A MORE NEARLY EXACT REPRESENTATION OF COOLING TOWER THEORY

Presented by:
Allen E. Feltzin
Airco Industrial Gases/
The BOC Group

Co-Author:
Dudley Benton
Tennessee Valley Authority
Division of Air & Water Resources

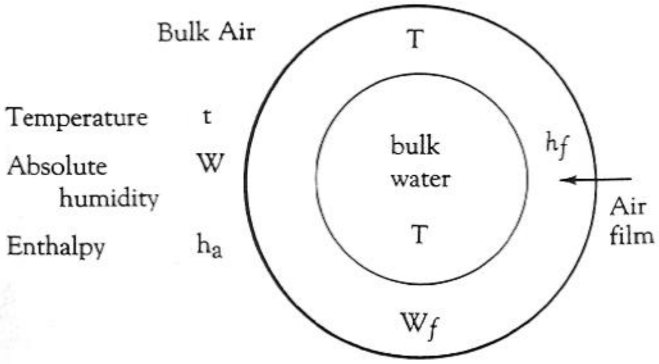
ABSTRACT

Merkel theory relied on several basic assumptions to simplify the mathematical calculations involved in the determination of KaV/L (cooling tower demand). Sixty-five years later, computers and numerical methods allow for more precise determinations. A description of both Merkel theory and the development of a more nearly exact representation are presented. The differences between the two methodologies are examined in detail. Necessary formulae and computational methods are presented in their entirety.

Historical Background: Merkel Theory

Merkel (1925), proposed a theory relating the evaporation and sensible heat transfer occurring in a direct contact process such as cooling of water or humidification of air, to an air enthalpy difference. Such a representation was suited (but not limited to) various types of cooling towers. The derivation was based on counterflow contact of water and air. In fact, there were six basic assumptions which were introduced at various points in the development to simplify the mathematics. As a starting point for this paper, it is desirable to present a relatively complete derivation of Merkel theory, so that the nature of these underlying assumptions and ultimate limitations of the model may be appreciated.

Merkel Theory



The model on which Merkel's theory was developed consists of a water droplet at temperature "T" surrounded by a thin air film (interface).

Assumption 1

The air film is saturated and therefore is also at temperature T.

It has a humidity W_f and an enthalpy h_f. Surrounding the air film is the bulk air mass at some lower temperature $t < T$, and absolute humidity $W < W_f$, and an enthalpy $h_a < h_f$.

If a is the interfacial surface (ft^2 / ft^3) and V is the contacting volume, (ft^3) then the interfacial surface area, $S=aV$ (ft^2) and the differential surface of the model droplet interfacial film is $dS = adV$ (surface element) L (lbs/hr-ft^2) of water are flowing downward and G (lbs/hr-ft^2) of air are flowing upward (counterflow).

Heat is transferred from the water droplet to the bulk air through the interface by two means, sensible heat transfer (convection) and latent heat of evaporation (mass transfer by diffusion).

Assumption 2

The interface offers no resistance to heat transfer from the water droplet to the bulk air by either of these mechanisms.

The sensible heat transfer rate by convection:

(1) $dq_s = K_c\,(T-t)\,adV$ where K_c is the conductive heat transfer coefficient, BTU/hr-ft^2°F.

The mass transfer rate is given by:

(2) $dL = K_p\,(P_T - P_t)\,adV$ where K_p is a diffusion coefficient. P_T and P_t are the partial pressures of water vapor in the interfacial film at temperature T and bulk air at temperature t respectively.

Assumption 3

Vapor content (absolute humidity) is proportional to partial pressure, i.e.

$$P_T \propto W_f$$

$$P_t \propto W$$

(3) $dL = K_M (W_f - W)\, adV$, K_M = mass transfer coefficient

The evaporative (latent) heat transfer rate due to diffusional mass transfer:

(4) $dq_L = \lambda\, dL = \lambda K_M (W_f - W)\, adV$; λ = latent heat of vaporization

and the total transfer rate is:

(5) $dq_{total} = [K_C (T-t) + \lambda K_m (W_f - W)]\, adV$

At this point in the derivation, the concept of humid heat, C_S (the heat capacity of an air-water vapor mixture) is usually introduced.
By addition and subtraction of a term $C_S (T-t)$ to the right hand side of equation (5) and algebraic manipulation (see Merkel: Appendix A), we arrive at:

(6)
$$dq_{total} = K_M \left\{ (C_S T + \lambda W_f) - (C_S\ t + \lambda W) + C_S (T-t) \left[\frac{K_c}{C_s K_m} - 1 \right] \right\} adV$$

Assumption 4

The Lewis relation, $Le = \frac{K_c}{C_S K_M} = 1$ for air-water vapor mixtures.

This causes the last term in equation (6) to vanish, and with it, K_c vanishes, so only one coefficient, the mass transfer coefficient, remains.

Assumption 5

The humid heat, C_S is assumed to be constant over the temperature range t to T. The term $C_S T + \lambda W_f$ is the enthalpy of the interfacial film, h_f, and the term $C_S + \lambda W$ is the enthalpy of the bulk air, h_a.

These two assumptions transform equation (6) to:

(7) $dq_{total} = K (h_f - h_a)\, adV$

We no longer need distinguish K as K_M

In performing a heat balance around the water droplet, it is apparent that the total heat content lost by the water droplet must be equal to the total heat content gained by the airstream.

(8) $dq_{total} = d (LC_w T) = d (Gh_a)$.

The mass flow of air (G) and the specific heat of water (C_w) are constant.

(9) $dq_{total} = C_w [LdT + TdL] = Gdh_a = K (h_f - h_a)\, adV$

Dividing by $(h_f - h_a)$ and integration yields

Continued on page 12

CTI WELCOMES NEW MEMBERS . . .

Albright & Wilson Americas
Andrew S. Bleadon, Delegate

Acquindustriali SRL
Bruno Neri, Delegate

Akzo Industrial Systems
Thomas R. Boys, President, Delegate

Bac Iberica, S.A.
Antonio Fernandez Casal, Affiliate Member

Baltimore Aircoil (Australia) Pty. Ltd.
Brian S. Drew, Affiliate Member

Baltimore Aircoil Int'l.
Jan A. Rombouts, Affiliate Member

Baltimore Aircoil Italy S.R.L.
Giacomo Moretti, Affiliate Member

Baltimore Aircoil Ltd.
Robert I. Macleod-Smith, Affiliate Member

BSC Industries Pte Ltd.
Tony Ng Keng Siang, Delegate

Calgon Vestal Laboratories
Thomas Douglas, Affiliate Member

Chiyoda Protech Corporation
Toshihisa Suzuki, Delegate

Cincinnati Gas Electric Co.
Donald R. Snider, Delegate

Clendenin Brothers, Inc.
William S. Marriott, Delegate

Cofimco SRL
Robert E. Mosiewicz, Delegate

Cooling Tower Components
Charles A. Peterson, Delegate

Dae Hwa Cooling Tower Industry Co., Inc.
Soo Hyun Cho, Delegate

Dae Hwa Cooling Tower Industry Co.
Philip Y. Cho, Affiliate Member

Davenport Cooling Technology, Inc.
Robert Fulkerson, Delegate

Decsa-De Cardenas Srl
Gianluca R. De Cardenas, Delegate

Decsa Iberica S.A.
Ramon Navarro, Affiliate Member

ESD Co. Ltd., S. B. Ban
Burger Assoc., Affiliate Member

The Falk Corporation
Suellen Torregrossa, Delegate

Foster Wheeler Power Products Limited
Brian Hartley, Delegate

Ingenieria y Desarrollo Industrial,
S. A., Delegate

Kipcon, Inc.
Mitchell H. Frumkin, Delegate

Marley Canadian Tower Asia Pacific
Pte Ltd, Affiliate Member

Continued on page 59

(10)

$$C_W\left\{\int_{T_{out}}^{T_{in}} \frac{L\,dt}{h_f-h_a} + \int_{T_{out}}^{T_{in}} \frac{T\,dL}{h_f-h_a}\right\} = KaV$$

$$= G\int_{h_{a_{in}}}^{h_{a_{out}}} \frac{dh_a}{h_f-h_a}$$

The details of the integration of KadV are given in Merkel: Appendix B.

Assumption 6

L = constant, implies:

1) $dL = 0$
2) L can be taken out of the integration sign

Therefore, dividing through by L,

(11)

$$\frac{KaV}{L} = C_W\int_{T_{out}}^{T_{in}} \frac{dT}{h_f-h_a} = \frac{G}{L}\int_{h_{a_{in}}}^{h_{a_{out}}} \frac{dh_a}{h_f-h_a}$$

Equation (11) is the integral representation of Merkel Theory. Note that the assumptions and simplifications made it possible to arrive at an ordinary 1st order, non-linear, separable differential equation (9). The non-linearity is the reason why an analytical function is not found as a solution. Instead, graphical or numerical methods must be employed.

The assumptions may be summarized as follows:

1. The interfacial film surrounding the water droplet is saturated and at water temperature T.
2. The interfacial film offers no resistance to heat or mass transfer from the water droplet to the bulk air.
3. The vapor content in the interfacial film and in the bulk air is proportional to the respective partial pressures.
4. The Lewis relation, $Le = \frac{K_C}{C_S K_M} = 1$.
5. The humid heat C_S is constant over the temperature range t to T.
6. Mass flow of water is constant.

Over the years, numerous approaches have been devised in an attempt to compensate for several of the above assumptions and approximations. Mickley (1949) introduced temperature and humidity gradients, heat and mass transfer coefficients from water to interfacial film, and from film to air. Baker and Mart (1952) developed a "hot water correction factor" which reduced the scatter in test data. Temperature correction factors have been applied to demand curves in some cases, and fill (characteristic) curves in others, a confusing situation at best. Effects of air temperature, barometric pressure, and salinity on fill characteristic KaV/L have been discussed by Lefevre (1985).

Integration Method

The integral for $\frac{KaV}{L}$ over water temperature,

(11a)

$$C_W\int_{T_{out}}^{T_{in}} \frac{dT}{h_f-h_a}$$

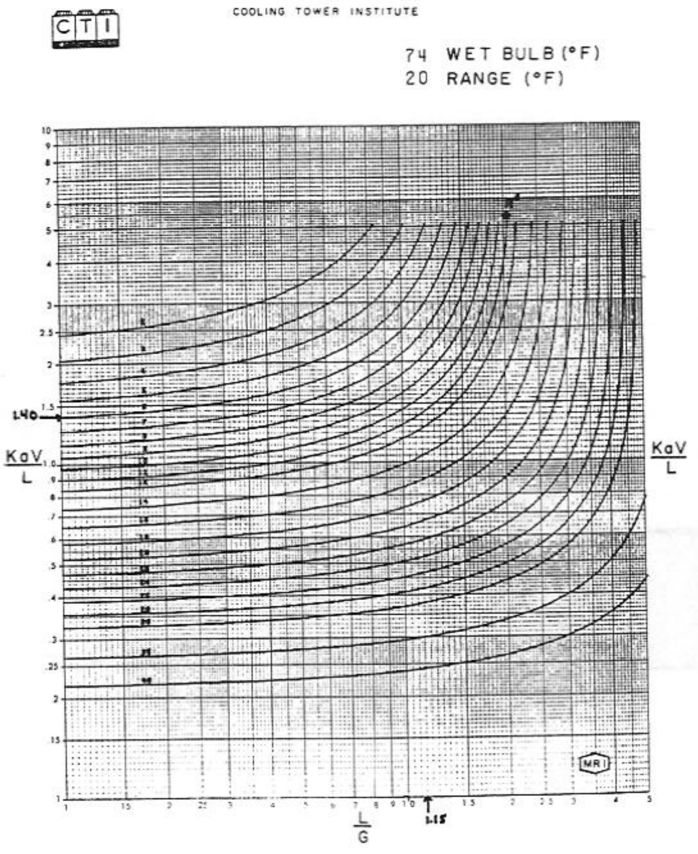
came to be associated with the number of *diffusion* units, while the integration over enthalpy,

(11b)

$$\frac{G}{L}\int_{h_{a_{in}}}^{h_{a_{out}}} \frac{dh_a}{h_f-h_a}$$

was known as the number of *transfer* units.

Historically, the diffusion unit format was favored over the transfer unit method, although, as noted by Lefevre (1987) either form is easy to integrate and the transfer unit method does not have the additional inaccuracy of neglecting the evaporation. The method of solving the integral was graphical, either by directly measuring the area under a plot of $\frac{1}{h_f-h_a}$ vs.



Water temperature, or by a quadrature method, in which the area under the curve is subdivided into incremental areas which correspond to successive increments in temperature. A detailed example of the quadrature method for a counterflow integration is given by Baker and Shryock (1961). The method of integration commonly employed has been the 4 point Chebyshev method, (CTI 1983), (Appendix C)

The Cooling Tower Institute (1967) using equation (11a) and the 4 point Chebyshev, expanded on earlier KaV/L vs. L/G "demand curves" such as those utilized by Foster Wheeler Corporation (1943) and J.F. Pritchard Company (1957). The CTI curves had the advantages of being computer generated and computer drawn, and made what had been very limited published data much more widely available and over a much broader selection of ranges and approaches. These have served the industry for over 20 years.

Assumption 7

The 4 point Chebyshev integration is sufficiently accurate.

Lefevre (1987) has noted the inaccuracies of 4 point Chebyshev integration. Benton (1989) in a comparison of seven methods, had found that for integrals of the type indicated in equation (11) Gauss quadrature was the most preferable method for computing cooling tower demand curves. Inaccuracies inherent in the Chebyshev method become more apparent at "pinch points" where the bulk air enthalpy line approaches very closely to the enthalpy line of the saturated interfacial film.

Development of A More Nearly Exact Representation

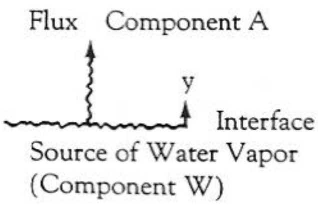
With modern computing techniques, and a selection of numerical integration methods available, most of these assumptions/approximations may be eliminated and the integration technique may be improved for greater accuracy.

For simplicity, all future references to "more nearly exact representation" in this paper are referred to as *exact*.

The following derivation, after Benton (1990) begins with first principles: For a binary mixture of non-reacting components, A and W, non-uniform composition, molecular diffusion within the mixture obeys Fick's first law. For steady state transfer in one dimension, this may be written in terms of the molal flux of component W in the y direction as:

(12)

$$\dot{m}_{w_y}'' = D_{AW} \left.\frac{dC_W}{dy}\right)_I$$



$$\dot{m}_{w_y}'' = \text{Molal flux } \frac{\text{lb - moles}}{\text{ft}^2 \text{ hr}}$$

D_{AW} = Diffusion coefficient (ft^2/hr)

$\frac{dC_W}{dy}$ = concentration gradient at the interface (lb-mole/ft^4)

One of the fundamental concepts of convective heat and mass transfer is that of a transfer coefficient. The underlying principles (viz. Fourier's law of conduction and Fick's law of diffusion) relate the slope of the transfer driving force (viz. temperature and concentration respectively) at an interface and the fluid property (viz. conductance and diffusion coefficient respectively) to the flux at the interface (viz. heat and mass respectively).

It is either impractical or impossible to measure the slope of the driving force at an interface without disturbing it. Furthermore, this interface is within a boundary layer, or a localized region of relatively abrupt change when compared to the whole flow field. For these and reasons of computational convenience, it is presumed that one can replace the slope at the interface and the fluid property with the difference in driving force across the entire boundary layer (i.e. from the interface out to the mean flow region) and a convective transfer coefficient.

$$k\, dT/dy \dashrightarrow h\,(T_I - T_\infty) \quad \text{or} \quad D\, dC/dy \dashrightarrow K\,(C_I - C_\infty)$$

More details of this as well as a schematic of a boundary layer can be found in the "Mass Transfer Coefficient" section of the chapter on "Mass Transfer of the ASHRAE Handbook of Fundamentals (1985).

In terms of a concentration difference at the interface and the bulk airstream, this can be expressed as:

$$(13) \quad \dot{m}_W'' = k\,(C_I - C_\infty)$$

k = Molal convective transfer coefficient

$(C_I - C_\infty)$ = Difference in concentrations

Converting concentrations to Mole Fractions:

$$(14) \quad \dot{m}_W'' = k\,\frac{(X_I - X_\infty)}{1 - X_I}$$

X_I = Mole fraction of water vapor at interface

X_∞ = Mole fraction of water vapor away from interface

1 = The mole fraction of water vapor on water vapor is 100%.

In the above expression the denominator must go to zero when the mole fraction of water vapor at the interface goes to 1 (which occurs at 212°F at 1 atmosphere) because there is no resistance to mass transfer at that condition - i.e. the air can receive an unlimited amount of water at 212°F.

Now that the molal flux has been expressed in terms of mole fractions, Dalton's law of partial pressures, may be invoked, which states:

Continued on page 15

"For an ideal mixture of ideal gases, the mole fractions are directly proportional to the partial pressures".

(15) $$\dot{m}_w'' = k\left(\frac{P_{WI} - P_{W\infty}}{P_{total} - P_{WI}}\right)$$

P_{WI} = Partial pressure of water vapor at interface (approximately the saturation pressure)

$P_{W\infty}$ = Partial pressure of water vapor away from interface

P_{total} = total pressure (ie. 14.7 psia)

In the above expression, it is apparent that when $T \rightarrow 212$, $P_{WI} \rightarrow 14.7$ and there is nothing to limit the transfer of water vapor into the air except inertia and the sonic speed (which are ignored in diffusion analyses).

Convective Mass Flux

For the purposes of this work, it is more convenient to define a mass convective transfer coefficient rather than a molal convective coefficient, and do so by the following expression:

(16) $$\dot{M}_W'' = K\left(\frac{X_I - X_\infty}{1 - X_I}\right)$$

$\dot{M}_W''$ = Convective mass flux $\left(\frac{\text{lbm}}{\text{hr ft}^2}\right)$

K = Convective mass transfer coefficient

X_I = Mass fraction of water vapor at interface

X_∞ = Mass fraction of water vapor away from interface

1 = mass fraction of water in water = 100%

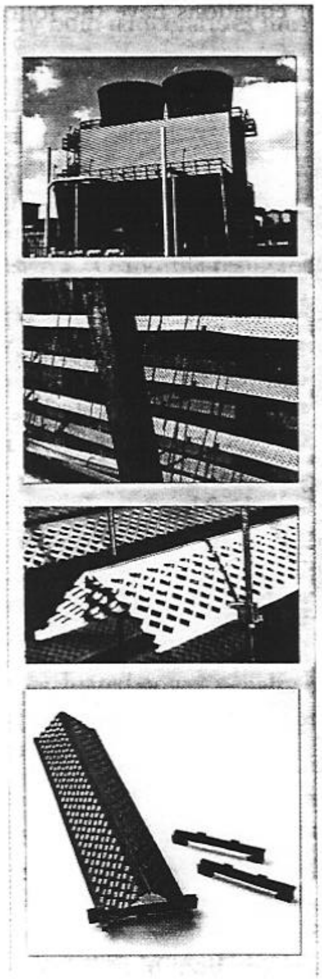
It is shown (Appendix D: Exact Derivation) that the mass fractions X_I and X_∞ may be expressed in terms of absolute humidities:

$$X_I = \frac{W_I}{W_I + 1} \qquad X = \frac{W_\infty}{W_\infty + 1_\infty}$$ and therefore

17. $$\dot{M}_W'' = K\left(\frac{W_I - W_\infty}{1 + W_\infty}\right)$$

When $T_I \rightarrow 212$, $W_I \rightarrow \infty$ so $\dot{M}_W''$ still goes to ∞ as it should.

If one neglects the resistance of the interface itself, then the mass transfer driving force is given by ($W_I - W_\infty$). A rigorous analysis of the interface results in a complex expression for the driving force which includes an exponential. The reader is directed to consult a mass transfer textbook for such details which are beyond the scope of this paper. The first order correction, however, will be introduced here which reduces the driving force by $1/(1+W_\infty)$, or $(W_I - W_\infty) / (1+W_\infty)$. This expression will always result in a smaller driving potential than the previous one.



ENGINEERED QUALITY

- We are the original innovators and manufacturer of Doron V-Bar/Fill.
- Optimum efficiency and refined engineering
- Fast delivery from inventory
- Celebrating over a quarter century of superior performance

ANJAC-DORON
PLASTICS, INC.

4466 N. Baldwin Ave. , El Monte, CA 91731 • (818) 579-5401

As was shown before for Merkel integration of KadV (Merkel: Appendix B)

(18) $$adV = (aV)\frac{dy}{Y}$$

According to mass transfer,

(19) $$\dot{M}_W''(aV)\frac{dy}{Y} = Gdw$$

(the mass of water lost is picked up by the airstream) using our result from the preceding section, noting that at the interface,

W_I = saturated = W_S ; $W_\infty = W$

(17) $$\dot{M}_W'' = K\left(\frac{W_S - W}{1 + W}\right)$$

and therefore,

(20) $$KaV\left(\frac{W_S - W}{1 + W}\right)\frac{dy}{Y} = Gdw$$

multiplying equation (20) by $\frac{L_{in}}{L_{in}}$ and dividing through by $Gd\left(\frac{y}{Y}\right)$

we may immediately write:

*(21) $$\frac{KaV}{L_{in}}\;\frac{L_{in}}{G}\;\left(\frac{W_S - W}{1 + W}\right) = \frac{dw}{d\left(\frac{y}{Y}\right)}$$

The total energy transfer rate (heat transfer due to mass transfer and convective heat transfer) is:

(22) $$\dot{Q}'' = K_M\left(\frac{W_S - W}{1 + W}\right)h_g + K_C(T_W - T_{db})$$

where h_g is the enthalpy of saturated water vapor and K_c is the convective heat transfer coefficient.

Again, invoking the Lewis relation, $K_C = K_M C_{p_a} Le$

(23) $$\dot{Q}'' = K\left[\left(\frac{W_S - W}{1 + W}\right)h_g + C_{p_a} Le\,(T_W - T_{db})\right]$$

(24) $$dq_{total} = KaV\left[\left(\frac{W_S - W}{1 + W}\right)h_g + C_p Le\,(T_W - T_{db})\right]\frac{dy}{Y} = Gdh_a$$

Multiplying (24) by $\frac{L_{in}}{L_{in}}$ and dividing through by $G\left(\frac{dy}{Y}\right)$, the following result is obtained:

*(25)

$$\frac{dh_a}{d\left(\frac{y}{Y}\right)} = \left(\frac{L_{in}}{G}\right)\left(\frac{KaV}{L_{in}}\right)\left[\left(\frac{W_S - W}{1 + w}\right)h_g + C_{p_a} Le\,(T_W - T_{db})\right]$$

By conservation of energy,

(26) $$dq_{total} = d\left[LC_W(T_W - 32)\right] = d(Gh_a)$$

Therefore,

*(27) $$\frac{d\left[\left(\frac{L}{G}\right)C_W(T_W - 32)\right]}{d\left(\frac{y}{Y}\right)} = \frac{dh_a}{d\left(\frac{y}{Y}\right)}$$

Also, from conservation of mass,

(28) $$dL = Gdw$$

This may be expressed as,

*(29) $$\frac{d\left(\frac{L}{G}\right)}{d\left(\frac{y}{Y}\right)} = \frac{dw}{d\left(\frac{y}{Y}\right)}$$

The asterisked equations (21), (25), (27), and (29) form a set of coupled, non-linear, ordinary differential equations.

While there are certainly more sophisticated and efficient techniques available, a Runge-Kutta numerical methods approach is quite applicable to this set of equations and gives a reasonable computation time. A computer algorithim for its implementation is given in Appendix E.

In integral form, the set of equations have the following form:

(30) Mass transfer:

$$W = W_{in} + \left(\frac{KaV}{L_{in}}\right)\left(\frac{L_{in}}{G}\right)\int_o^y \left[\left(\frac{W_s - W}{1 + W}\right)\frac{dy'}{Y}\right.$$

(31) Heat transfer:

$$h_{a_{out}} = h_{a_{in}} + \left(\frac{KaV}{L_{in}}\right)\left(\frac{L_{in}}{G}\right)\int_o^y \left[\left(\frac{W_s - W}{1 + W}\right)h_g + C_{p_a} Le\,(T_w - T_{db})\right]\frac{dy'}{Y}$$

(32) Conservation of Energy:

$$T_w = 32 + \frac{C_w\left(\frac{L}{G}_{in}\right)(T_{cold} - 32) + (h_a - h_{a_{in}})}{\frac{L}{G}\;C_w}$$

(33) Conservation of Mass:

$$\frac{L}{G} = \left(\frac{L_{out}}{G}\right) + W - W_{in}$$

Equations (30) and (31) must be integrated simultaneously as the results of one are required in order to evaluate the other. The Runge-Kutta technique is especially convenient for doing this simultaneous integration. In addition, Equations (32) and (33) must be solved algebraically at each step of integration and the various properties must be evaluated.

Equation (31) may be written explicitly in terms of $\frac{KaV}{L}$ as

(34)

$$\frac{KaV}{L_{in}} = \frac{h_{a_{out}} - h_{a_{in}}}{\frac{(L_{in})}{G}\int_{o}^{y}\left[\left(\frac{W_s - W}{1+W}\right)h_g + C_{p_a}\,Le\,(T_w - T_{db})\right]\frac{dy}{Y}}$$

Equation (34) forms the basis of the exact representation of $\frac{KaV}{L}$.

COMPARISON OF EXACT VS MERKEL

Application Examples

I. Mechanical Draft Cooling Towers

The quantity $\frac{KaV}{L}$ represented by equation (11), Merkel, or equation (34), Exact, has been applied to both cooling tower demand curves and fill (packing) characteristic curves. The latter are generated by application of the same mathematical techniques to a set of test data for a given packing over a range of test conditions. To use the exact method, inlet and exit dry bulb temperatures must be recorded.

Using a characteristic curve generated for counterflow film fill superimposed on a set of demand curves, a design (or operating) point was calculated for a specific condition. This "point" consisted of an L/G and a KaV/L required for a certain approach at a given wet bulb and range. This lends itself to a comparison of Merkel vs. Exact in terms of a shift in design L/G and the resultant effect on fan horsepower.

Suppose the following performance were specified: 101/91/78. It turns out that 5′ of film fill is required. Using the particular fill data for 12 mm fill, it is seen that the intersection point for Merkel is L/G = 3.03 while the more exact solution methods give L/G = 2.94 (Figures 1 & 2). This is a ratio (Merkel: Exact) of 1.0306. This means, according to the normal fan laws, that the more exact value for L/G would have to allow for 9.5% more fan horsepower to do the required job (tower size held constant). Another way of looking at it is that the Merkel designed tower at its best would test at 97% capability.

At an evaluated power penalty of $2,500 per HP on a 5-cell tower using 200 HP fans, the horsepower deviation could be worth $250,000.

At a second condition, corresponding to 135/88/78 (chosen to investigate a hot water effect), the Merkel L/G was 1.57, the exact value for L/G was 1.59. Note, this is a shift in the *opposite* direction, the ratio (Merkel: Exact) = 0.987. A Merkel tower would be overdesigned by 3.9% on fan horsepower (Figs. 3 & 4).

101/91/78 MERKEL L/G = 3.03

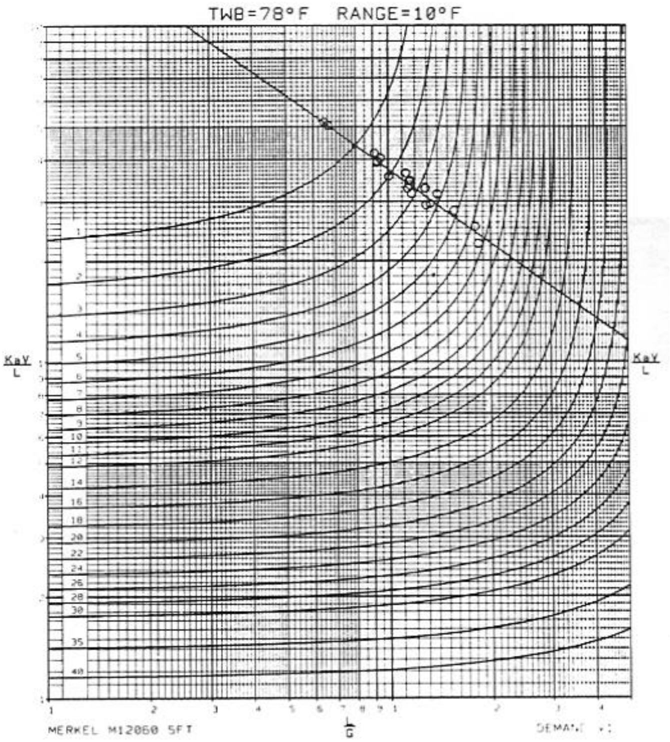


Figure 1

101/91/78 EXACT L/G = 2.94

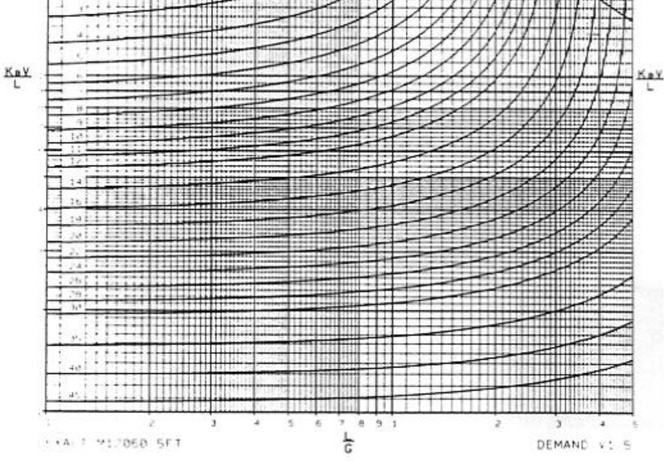


Figure 2

Continued on page 19

135/88/78 MERKEL L/G = 1.57

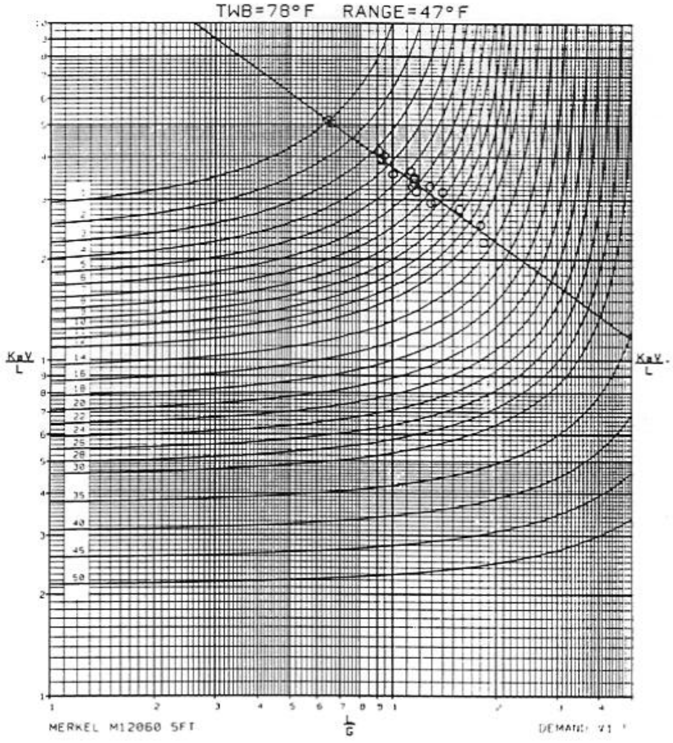


Figure 3

135/88/78 EXACT L/G = 1.59

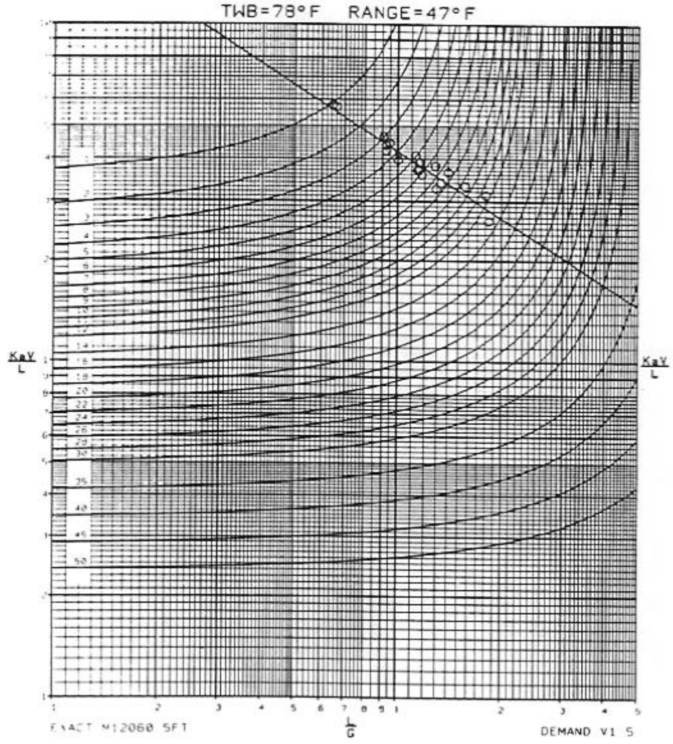


Figure 4

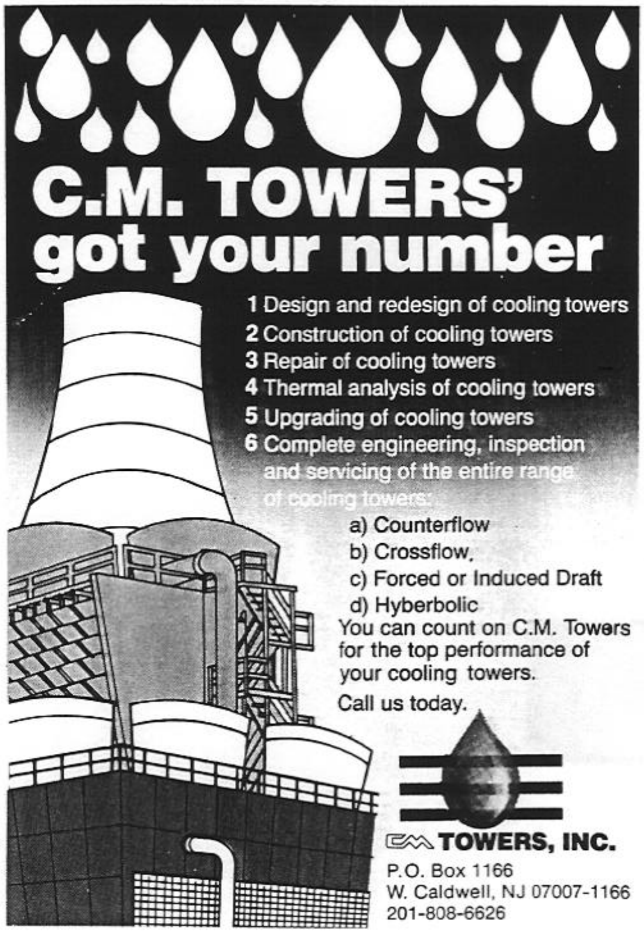
Thus, it has been demonstrated that the difference between Merkel and the exact solution will give changes of the order of 1-3% in the design L/G which are significant, especially when considered in terms of fan horsepower (3-9%), and that the changes can be in either direction depending on the conditions selected. This probably has to do with the range of applicability of extrapolation for test data reduced by the Merkel method, as well as the inherent scatter in the data itself. A strong cautionary note is to avoid using "mixed" methods.

A Merkel characteristic curve superimposed on corrected demand curves (whether by exact method or an empirical adjustment) or corrected characteristic curves imposed on Merkel demand curves will lead to errors as much as an order of magnitude greater than the effects seen when consistent methods are applied to both fill and demand curves together. This represents a very real danger until the industry adopts a single, consistent method.

II. Natural Draft Towers

Saturation vs. Unsaturation: Effect on Density and Natural Draft Calculations.

It is common to assume that the inlet air can be characterized solely by the wet-bulb temperature. While the wet-bulb for all practical purposes does define the enthalpy, it does not completely describe the thermodynamic state of



C.M. TOWERS' got your number

1 Design and redesign of cooling towers
2 Construction of cooling towers
3 Repair of cooling towers
4 Thermal analysis of cooling towers
5 Upgrading of cooling towers
6 Complete engineering, inspection and servicing of the entire range of cooling towers:
  a) Counterflow
  b) Crossflow,
  c) Forced or Induced Draft
  d) Hyberbolic

You can count on C.M. Towers for the top performance of your cooling towers.
Call us today.

CM TOWERS, INC.
P.O. Box 1166
W. Caldwell, NJ 07007-1166
201-808-6626

the air. At the saturation point, the wet- and dry-bulb temperatures are identical. Thus, characterizing the inlet air by the wet-bulb alone is equivalent to assuming that the inlet air is saturated. In the analysis of counterflow cooling towers, the two areas where the assumption that the incoming air is saturated has the greatest effect is on the density, and the sensible heat transfer.

In the case of natural draft towers, the density difference, or buoyancy, is the driving force for the airflow. This "thermal draft" is approximately equal to the tower height multiplied by the difference in density between the outside ambient air and the inside warm, humid air.

For example, for a given airflow, which, at constant heat load, means a constant exit wet bulb, the draft is the lowest when the exit air is saturated. When the air is not saturated, the air is actually warmer and lighter inside the tower and the difference between ambient and inside air is larger, hence more draft and more airflow.

Draft = ht. x ($D_{ambient} - D_{exit}$); D=density

Fig. 5 shows the effect of relative humidity on density of moist air. Fig. 6 indicates how required stack height is affected by exit relative humidity in a natural draft tower. Fortunately, it is exceedingly rare that exit relative humidities in a natural draft tower of < 95% are attained, so that deviations from Merkel theory are again on the order of several percent. In practice, empirical correction factors have been applied to Merkel theory. Fig. 7 shows the effect on tower approach. In a utility, a valuation of $1,000,000

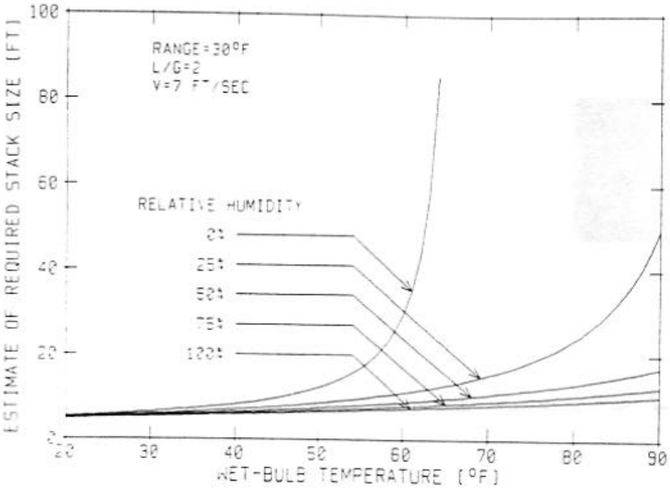


Figure 6 — *Stack size required to provide one velocity head of draft*

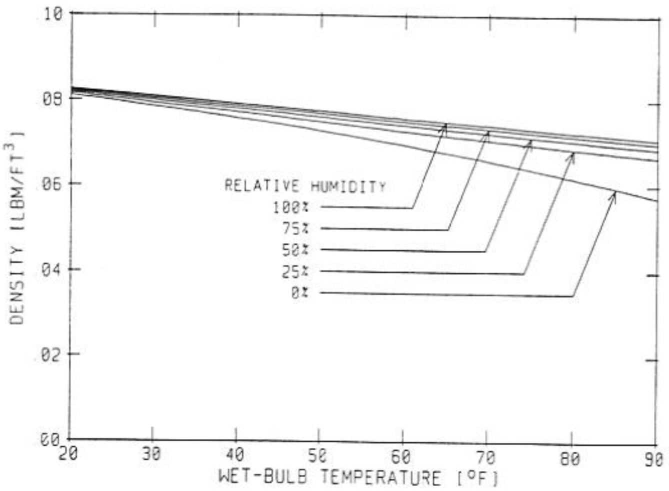


Figure 5 — *Density of moist air at 1 atmosphere*

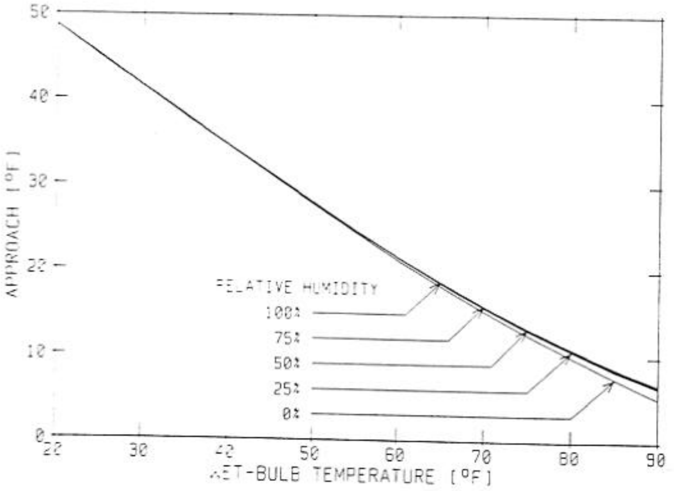


Figure 7 — *Typical approach as a function of TWB and RH*

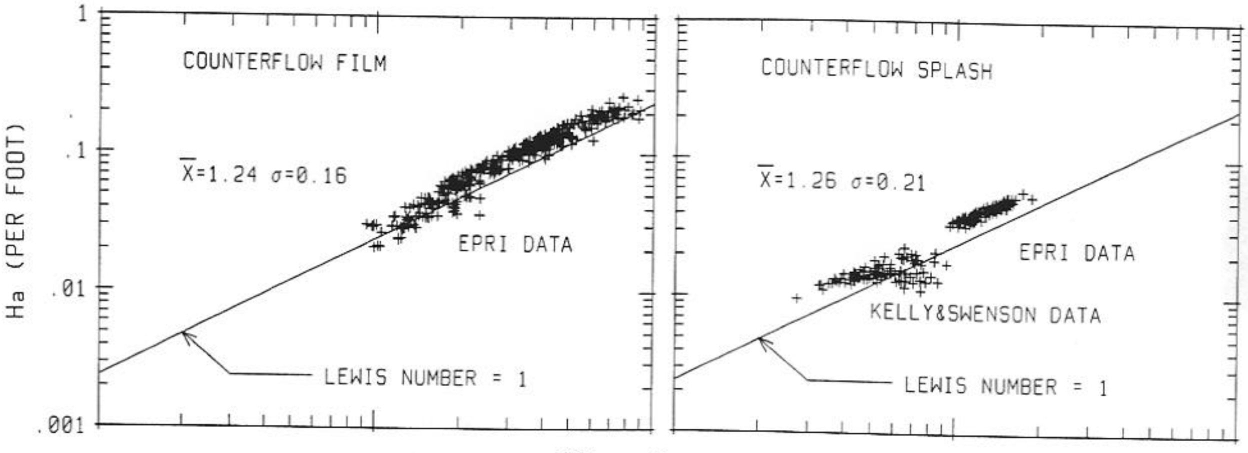


Figure 8

per degree is not inappropriate, so even a deviation in performance of several tenths of a degree is quite significant.

III. Lewis Relation

As mentioned earlier, Merkel theory is predicated on the assumption that the Lewis relation is numerically 1.0. Exact theory allows the Lewis relation to be treated as an input value. A recent analysis of both splash and film packings (Fig. 8) by Benton indicates that for counterflow, a Lewis value of 1.25 is more appropriate. It is interesting that this does not appear to be dependent on whether the packing is splash type or film type, but only on the configuration (counterflow).

Lefevre (1990) provided information regarding the influence of the Lewis relation as illustrated in the following table, where both calculations were exact, only using different input values for the Lewis relation:

Lewis Relation # = 1.00
Range 18°F

		Exact	Humidity		
Tal	F	112.0	99.1	94.1	90.2
Rhl	%	8	10	15	20
Twbl	F	67.7	62.6	62.6	62.6
Ta2	F	102.4	93.8	90.7	88.3
Rh2	%	97.2	96.3	96.0	95.9
Twl	F	87.6	79.7	77.1	75.2

Influence of Lewis number
Lewis Relation # = 1.25
Range 18°F

		Exact	Humidity		
Tal	F	112.0	99.1	94.1	90.2
Rhl	%	8	10	15	20
Twbl	F	67.7	62.6	62.6	62.6
Ta2	F	102.6	94.0	90.9	88.6
Rh2	%	95.9	94.7	94.3	94.0
Twl	F	87.4	79.5	76.9	74.9

Using a Lewis relation equal to 1.25 results in approximately 0.2°F lower cold water, due to a predicted exit dry bulb 0.2°F higher, a relative humidity 2% lower, resulting in more draft and more airflow.

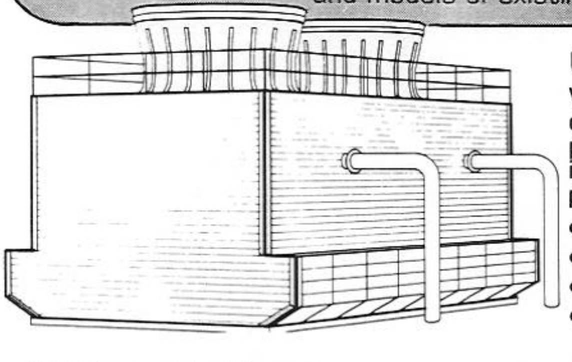
Conclusion

Somewhat strangely, the industry is, at present, in a situation similar to that addressed by Lichenstein (1943) when he initially proposed acceptance of Merkel theory by the industry almost fifty years ago. In his prophetic words:

> "The cooling tower industry should, therefore, be able now to standardize and adopt a common basic theory which would be accepted for correlation of its vast amount of experimental data and practical experience, and which could also be used to calculate performance at the guarantee point from test data taken under other conditions of service."

Tower Performance, Inc.
The Cooling Tower Specialists!

Since 1964, Tower Performance has been providing full cooling tower service by constructing new towers and upgrading and repairing all makes and models of existing cooling towers.



Upgraded Cooling Towers

We can upgrade the cooling capacity of existing towers to provide colder water. The increased efficiency yields fast payback. Our services include:

- Repair
- Replacement
- Overhaul
- Emergency Repairs
- Scheduled Maintenance
- Inspection/ Evaluation

New Cooling Towers

With our custom design, we can handle special applications for all industries. Our counterflow towers are energy efficient with low fan horsepower and pumping head. We specialize in building new cooling towers on existing basins.

Our counterflow design minimizes ice damage during winter.

Nationwide Service We supply **replacement parts** for **all** cooling towers - call or fax our nearest office.

TOWER PERFORMANCE, INC.

CTI
MEMBER

387 Passaic Avenue
Fairfield, NJ 07004
(201) 227-5016 / (212) 355-0746
Fax (201) 227-8605

4031 Broadway
Houston, TX 77087
(713) 643-0691
Fax (713) 643-0310

P.O. Box 77906
Baton Rouge, LA 70879
(504) 346-7786
Fax (504) 293-7461

TPI

We are again at a crossroads, where many different approaches to empirical correction factors have been taken, resulting in some confusion and lack of standardization in the industry. As testing methods become more refined, inherent scatter in data will be reduced, creating the need for exact determinations. A sound theoretical base is needed to extend applications to higher salinities, elevated water temperatures, etc. without the need for adding additional correction factors to those already employed. We respectfully propose that the methods put forth in this paper be critically examined, and, if found satisfactory, accepted for general use.

The authors of this paper wish specifically to acknowledge contributions from M. Lefevre (MRL Corporation), R. Harrison (Baltimore Aircoil), and R. Des Jardins (Des Jardins Associates) as well as to the members of the Non-Merkel Methods Committee in general.

NOMENCLATURE

Symbol	Designation	Units	
a	Interfacial surface	ft^2/ft^3	
C	Specific heat	BTU/lb°F	distinguished by context
C	Molal concentration	lb-mole/ft^3	
C_S	Humid heat	BTU/lb°F	
d	Differentiation operator		
D	Diffusion coefficient	ft^2/hr	
G	Air mass flow rate	lbs/hr-ft^2	
h	Enthalpy	BTU/lb	
k	Molal convective transfer coefficient		
K	Generalized transfer coefficient		
Le	Lewis Relation $\frac{K_C}{C_S K_M}$		
L	Water mass flow rate	lbs/hr-ft^2	
$\dot{m}$	Molal flux	lb-moles/ft^2 – hr	
$\dot{M}$	Convective mass flux	lbm/hr-ft^2	
P	Partial pressure	lb/ft^2	
q	Heat transfer rate	BTU/hr	
$\dot{Q}$	Total energy transfer rate	BTU/hr	
Rh	Relative humidity		
S	Interfacial surface area	ft^2	
t	Air temperature	°F	
T	Water temperature	°F	
V	Contacting volume	ft^3	
wb	Wet bulb		
W	Humidity	lbs water/lb dry air	
X	Mole fraction		

Subscripts

a, A	Air	
c	Convective/conductive heat transfer (depending on context)	
db	Dry bulb	
f	Interface or film	
g	Saturated water-vapor	
I	Property at interface	
L	Latent heat transfer	
M	Mass transfer	
p	Diffusional mass transfer	
s	Sensible heat transfer	
S	Saturated	
t	Bulk air temperature	°F
T	Water temperature	°F
w,W	Water	
∞	Property far from interface	

Greek Letters

λ	Latent heat of vaporization	BTU/lb
$\propto$	Proportional to	

REFERENCES

Merkel, F. Verdunstungskulung, V.D.I. Forschungsarbeiteh, No. 275, Berlin, 1925.

Mickley, H.S. Chemical Engineering Progress Vol. 45, 1949, p. 739.

Baker, R. and Mart, L.T., Refrigeration Engineering 1952 p. 965.

Lefevre, M., CTI Paper 85-8 New Orleans, La., Jan. 21-23 1985.

Lefevre, M., "Eliminating the Merkel Theory Approximations..." Journal of the CTI Vol. 8, No. 1, pp. 36-44, 1987.

Baker & Shryock, Journal of Heat Transfer, April 1961 Appendix B and Table 1B.

Chebyshev 4 Point Integration Procedure - CTI Bulletin ATC-105 using Form ATC-127, 1983.

CTI Blue Book of Counterflow Demand Cooling Curves 1967 CTI, Houston, Tx, 1967.

Cooling Tower Performance: Bulletin CT-43-2, Foster Wheeler Corporation, 1943.

Counterflow Cooling Tower Performance J.F. Pritchard Company of California, 1957.

Benton, D., Letter to CTI Non-Merkel Committee Members, 7/24/89.

Benton, D., Unpublished notes, CTI Summer Meeting, Ashville, N.C. July 9-11, 1990.

ASHRAE Handbook: Fundamentals; American Society of Heating, Refrigerating, and Air Conditioning Engineers, New York, 1985.

Lefevre, M., private communication, 8/19/90.

Lichenstein, J., Trans ASME, 1943 p. 779.

Continued on page 24

APPENDIX A
Merkel Derivation

Starting With
(5)

$$K_C[(T-t)+\lambda K_M(W_f-W)]\ adV$$

$$= K_C\ T\ adV - K_C t\ adV + \lambda K_M\ W_f\ adV - \lambda K_M\ W\ adV$$

$$= K_M\left[\lambda W_f - \lambda W + \frac{K_c}{K_M}\ T - \frac{K_c}{K_M}\ t\right] adV$$

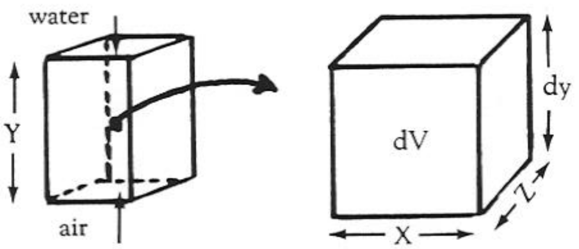
add and subtract $C_S\ (T-t)$

$$K_M\left[\lambda W_f + C_S\ (T-t) - \lambda W + \frac{K_C}{K_M}\ (T-t) - C_S\ (T-t)\right] adV$$

$$= K_M\left\{(C_S\ T + \lambda W_f) - (C_S t + \lambda W) + C_S\ (T-t)\left[\frac{K_c}{C_S\ K_M} - 1\right]\right\} adV$$

APPENDIX B
Merkel Derivation

Let's consider the volume element dV in more detail: It is intuitively obvious that when the integrations are performed, dV becomes V. However, dV = XZdy in cartesian coordinates,



so the integration is really being performed in one dimension, not three.

$$V = XYZ,\ \text{so}\ XZ = \frac{V}{Y}\ ;\ dV = XZdy = \frac{V}{Y}dy$$

$$\int_0^V KadV' = \int_0^y Ka\ \frac{V}{Y}\ dy' = \frac{KaV}{Y}\int_{y=0}^{y=Y} dy'$$

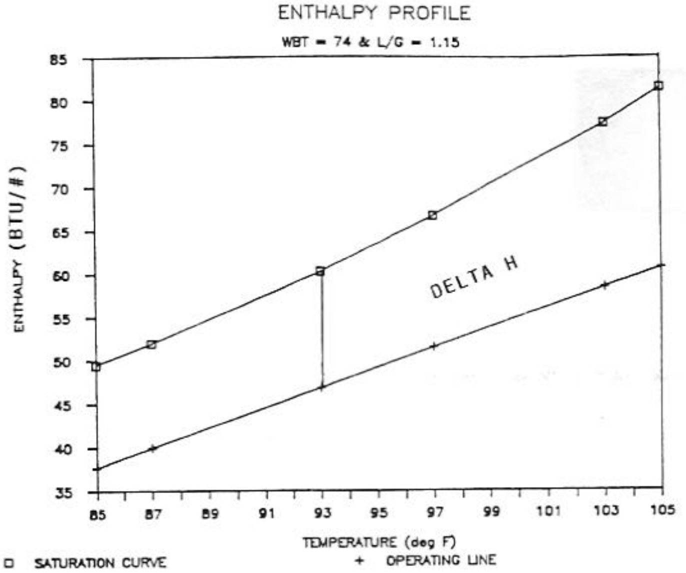
Y = a constant

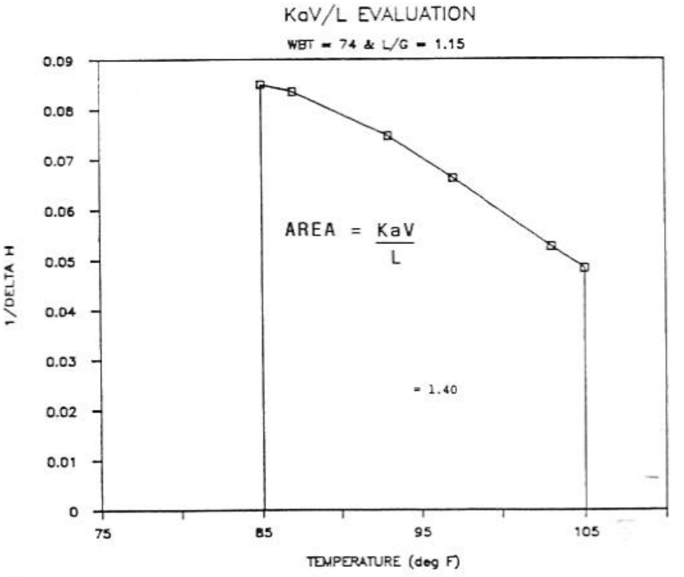
Therefore the integration yields $\frac{KaV}{Y}\cdot Y = KaV$

Note: Ka is assumed to be constant or represented by some average value over the fill volume.

APPENDIX C

REDUCTION TO PRACTICE
Example Calculation by Merkel Theory using graphical integration





APPENDIX D

Exact Derivation

Use of Humidity

$$X = \frac{\text{mass of water vapor}}{\text{total mass}} \qquad W = \frac{\text{mass water vapor}}{\text{mass dry air}}$$

$$\mathbf{W+1} = \frac{\text{Water}}{\text{Air}} + \frac{\text{Air}}{\text{Air}} = \frac{\text{Water} + \text{Air}}{\text{Air}}$$

$$\frac{1}{W+1} \qquad \frac{\text{Air}}{\text{Water} + \text{Air}}$$

$$1-\frac{1}{W+1} = \frac{W+1}{W+1} - \frac{1}{W+1} = \frac{W}{W+1} - \frac{\text{Water} + \text{Air}}{\text{Water} + \text{Air}} - \frac{\text{Air}}{\text{Water} + \text{Air}} = \frac{\text{Water}}{\text{Water} + \text{Air}} = X$$

$$\frac{W}{W+1} = X$$

$$\frac{X_I - X_\infty}{1 - X_I} = \frac{\frac{W_I}{1+W_I} - \frac{W_\infty}{1+W_\infty}}{1 - \frac{W_I}{1+W_I}} = \frac{\frac{W_I\,(1+W_\infty) - W_\infty\,(1+W_I)}{(1+W_I)\qquad(1+W_\infty)}}{\frac{1+W_I}{1+W_I} - \frac{W_I}{1+W_I}}$$

$$= \frac{\left[\frac{(W_I + W_I\,W_\infty - W_\infty - W_I\,W_\infty)}{(1+W_I)\ (1+W_\infty)}\right]}{\left[\frac{(1+W_I - W_I)}{(1+W_I)}\right]} = \frac{\left[\frac{(W_I - W_\infty)}{(1+W_I)\ (1+W_\infty)}\right]}{\left[\frac{1}{(1+W_I)}\right]} = \frac{W_I - W_\infty}{1 + W_\infty}$$

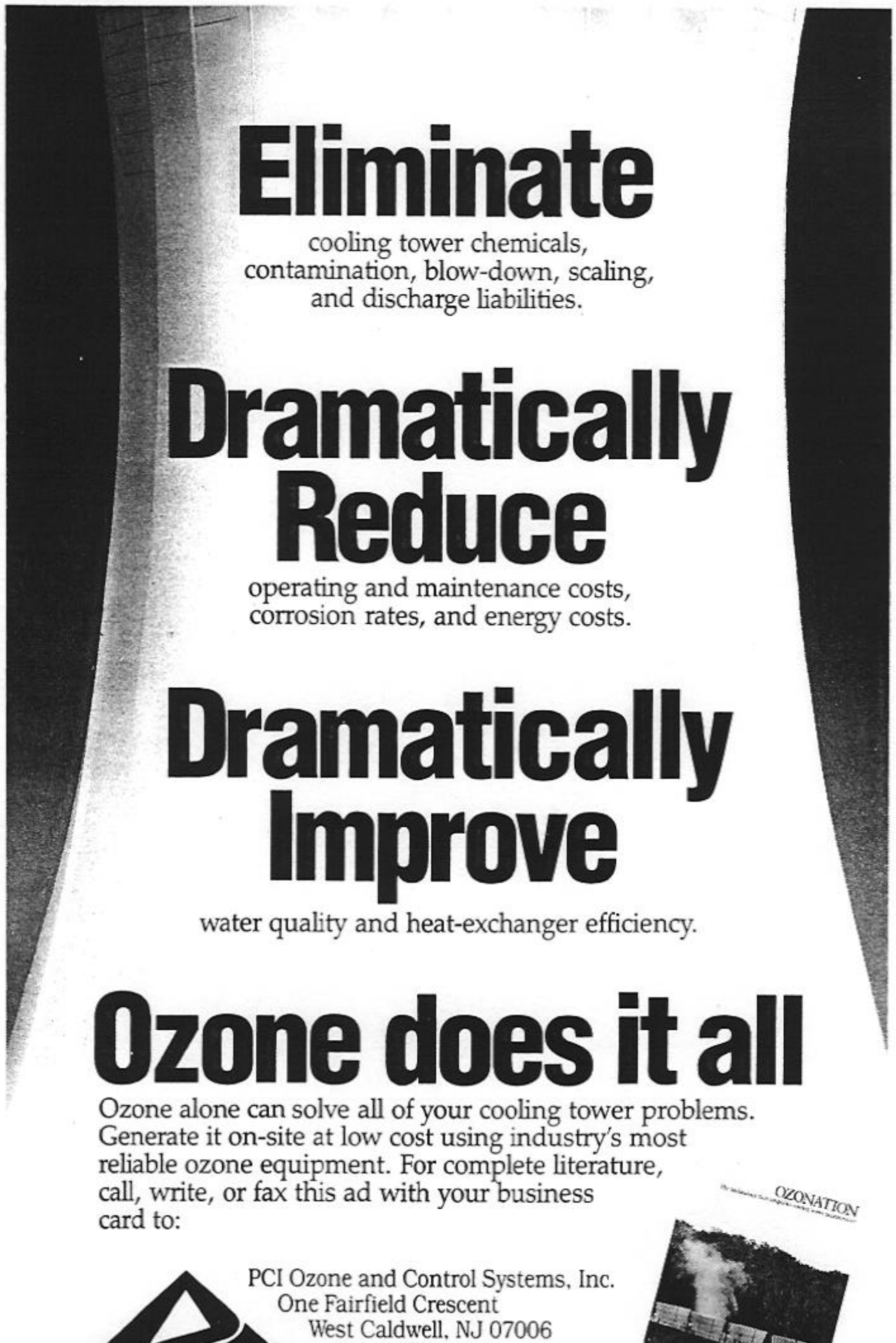
APPENDIX E: EXACT
Algorithm for Runge-Kutta Method

```
      SUBROUTINE RK4
        (DYDX, X, DX, Y, DY, N, W, Y)
C
c     perform one step of 4-th order Runge-Kutta
        integration
C
C     definition of variables:
C
C     X ................. the independent variable (scalar)
C     DX .......................the step size of X (scalar)
C     Y ......... the dependent variable (vector or array)
C     DY........................ dY/dX (vector or array)
C     DYDX ......... name of subroutine defining dY/dX
C     N ............... number of the dependent variables
C     W .............. working space of dimension (N, 4)
C     V .................. working space of dimension (4)
C
C     NOTE: the differential must be provided by
        subroutine DYDX (X, Y, DY)
C
      IMPLICIT INTEGER*2 (I-N), REAL*4
        (A-H, O-Z)
      DIMENSION Y(N), DY(N), W(N, 4), V(N),
        A(3), B(4)
      DATA A/ .5,.5,1./
      DATA B/.166666667, 2*.333333333,.166666667/
C
      XX=X
      DO 100 I=1, N
      100 V(I)=Y(I)
C
      DO 120 ISTEP=1, 4
      IF(ISTEP.EQ.1) 60 TO 120
      ISTEP1=ISTEP-1
      XX=X+DX*A (ISTEP1)
      DO 110 I=1, N
      DY(I)=A(ISTEP1)*W(I,ISTEP1)
      110 V(I)=Y(I)+DX*DY(I)
C
      120 CALL DYDX(XX, V, W(1, ISTEP))
C
      DO 140 I=1, N
      DY(I)=0.
      DO 130 J=1.4
      130 DY(I)=DY(I)+B(J)*W(I.J)
      140 Y(I)=Y(I)+DX*DY(I)
      X=X+DX
C
      RETURN
      END
```

About the Authors:

A. E. Feltzin is manager of utilities for Airco Industrial Gases, Murray Hill, NJ. He earned a B.S. in chemistry from the University of Delaware, 1972. Current responsibilities include cooling tower performance and design, heat exchanger construction and design, and water treatment program specifications. Al has authored/co-authored 8 technical papers and is a member of NACE, ASME, and CTI.

Dudley J. Benton received a B.S. (focusing on fluid mechanics) and M.S. (focusing on numerical analysis of heat exchangers in offshore applications) in Ocean Engineering from Florida Atlantic University in Boca Raton, Florida in 1976 and 1977 respectively. He received a Ph.D. (focusing on experimental and numerical studies of ultra-high temperature boiling) from the University of Tennessee in 1982. He has worked at the TVA Engineering Laboratory in Norris, Tennessee since 1980 where he is the staff specialist in numerical analysis of thermodynamics and combined heat and mass transfer.


Eliminate
cooling tower chemicals,
contamination, blow-down, scaling,
and discharge liabilities.

Dramatically Reduce
operating and maintenance costs,
corrosion rates, and energy costs.

Dramatically Improve
water quality and heat-exchanger efficiency.

Ozone does it all

Ozone alone can solve all of your cooling tower problems. Generate it on-site at low cost using industry's most reliable ozone equipment. For complete literature, call, write, or fax this ad with your business card to:

PCI Ozone and Control Systems, Inc.
One Fairfield Crescent
West Caldwell, NJ 07006
(201) 575-7052
FAX (201) 575-8941

PCI OZONE & CONTROL SYSTEMS, INC.

CALL FOR PAPERS, 1992 ANNUAL MEETING, FEBRUARY

Papers are being considered for presentation at the 1992 Annual CTI Meeting, to be held February 3-5, at the Wyndham Greenspoint Hotel in Houston, Texas.

Contact the CTI office for an Abstract Form. If you have new, unpublished information to report that would be of interest to the cooling tower industry, this is an opportunity to present it to the only forum devoted entirely to cooling towers.

For further information contact the CTI office, 713/583-4087.

Program Chairmen: Jack Bland, ChemTreat, Inc. and John Mangelinkx, Conoco.