A Simplified Approach for Modeling a Thermal Plume in a Stratified Ambient

a report to the

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by

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The entrainment for a buoyancy-dominated thermal plume can be estimated from the following equation:

$$\frac{Q_E}{Q_D} = \alpha \left(\frac{\rho_A - \rho_D}{\rho_A} \right)^{\frac{1}{3}} \left(\frac{D}{W} \right)^{\frac{3}{5}} \tag{1}$$

where Q_D and Q_E are the discharge and entrained flows, α is the entrainment coefficient (an empirically-derived constant), ρ_A and ρ_D are the ambient and discharge density, D is the depth, and W is the width. This equation assumes constant ambient conditions. An equation for variable ambient conditions can be obtained by considering a differential equation with a power-law relationship for the entrained flow:

$$\frac{d\,Q_E}{dZ} = \beta \left(\frac{\rho_E - \rho_P}{\rho_E} \right)^{\frac{1}{3}} Z^n \tag{2}$$

where ρ_E and ρ_P are the entrained and plume density and Z is the vertical distance. For a uniform ambient the density difference is approximately inversely proportional to the total flow. As the plume is diluted by the ambient, the difference in density between the plume and the ambient diminishes proportionally, which invites the following substitution:

$$\frac{d\,Q_E}{dZ} = \beta \frac{Z^n}{\left(Q_D + Q_E \right)^{\frac{1}{3}}} \tag{3}$$

This ordinary separable differential equation can be solved for the dilution:

$$1 + \frac{Q_E}{Q_D} = \left[1 + \beta \left(\frac{Z}{W} \right)^{(n+1)} \right]^{\frac{3}{4}} \tag{4}$$

In order for the dilution to be proportional to the three-fifths power of the depth, n must equal minus one-fifth. Further comparison implies:

$$1 + \frac{Q_E}{Q_D} = \left[1 + \alpha^{\frac{4}{3}} \left(\frac{\rho_A - \rho_D}{\rho_A} \right)^{\frac{4}{9}} \left(\frac{D}{W} \right)^{\frac{4}{5}} \right]^{\frac{3}{4}} \tag{5}$$

The separable differential equation can be written in integral form:

$$5W^{\frac{4}{5}} \int_{Q_D}^{Q_D+Q_E} (Q_D+Q)^{\frac{1}{3}} d\left(Q_D+Q\right) = 3\,\alpha^{\frac{4}{3}} \left(\frac{\rho_A - \rho_D}{\rho_A}\right)^{\frac{4}{9}} Q_D^{\frac{4}{3}} \int_0^D \frac{dZ}{Z^{\frac{1}{5}}} \qquad \textbf{(6)}$$

The conservation of energy can also be expressed in integral form:

$$CW \int_{T_D}^{T_P} \rho_P \left(Q_D+Q_E\right) dT = C \int_0^D \rho_E\, Q_E T_E\, dZ \qquad \textbf{(7)}$$

where C is the specific heat. For a stratified ambient the density and temperature will vary as the plume rises and entrains water. Any of a number of numerical integration techniques can be used to solve these simultaneous integral equations. The fourth-order Runge-Kutta method is used here. A comparison between field data, two zero-dimensional TVA models, ZZ's three-dimensional model, and ZZ's one-dimensional model is shown in the following figure.

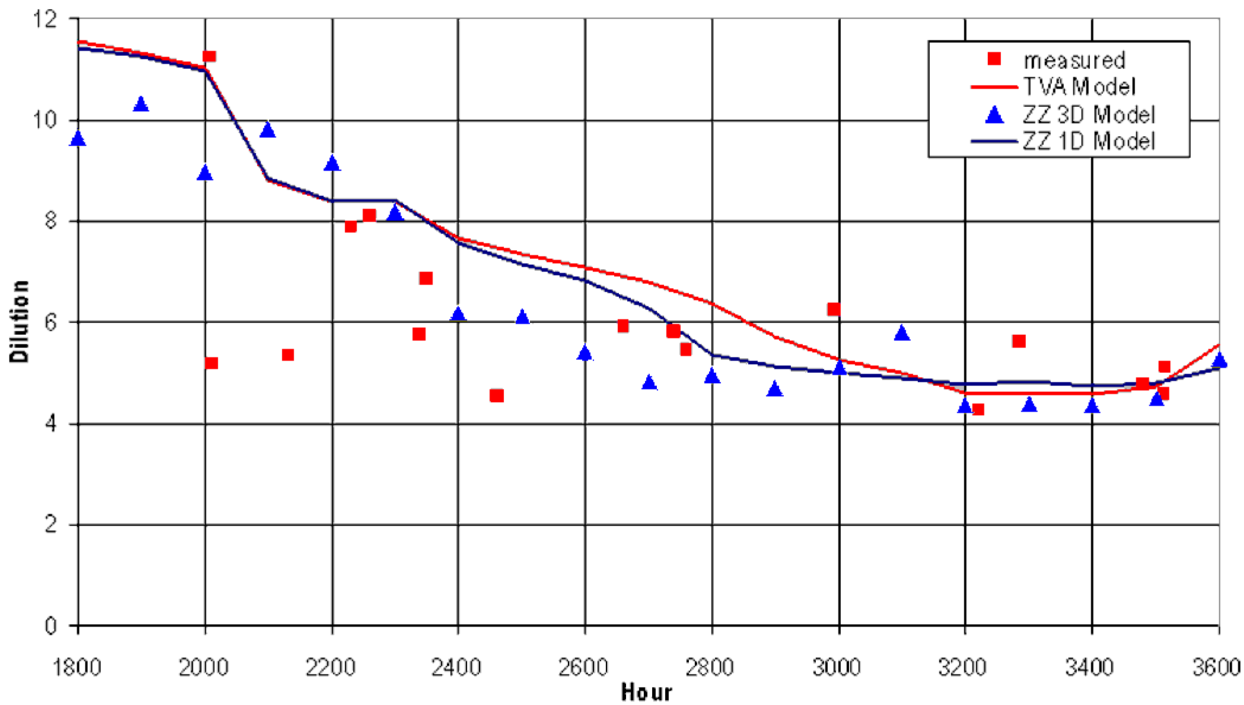

Figure 1. ZZ Model Results and 1977 Field Data

These calculations are based on a uniform upstream temperature, in which case, the current one-dimensional plume model closely follows the TVA zero-dimensional model. The increase in dilution with depth as computed using the two-dimensional Sequoyah plume model is shown in the following figure.

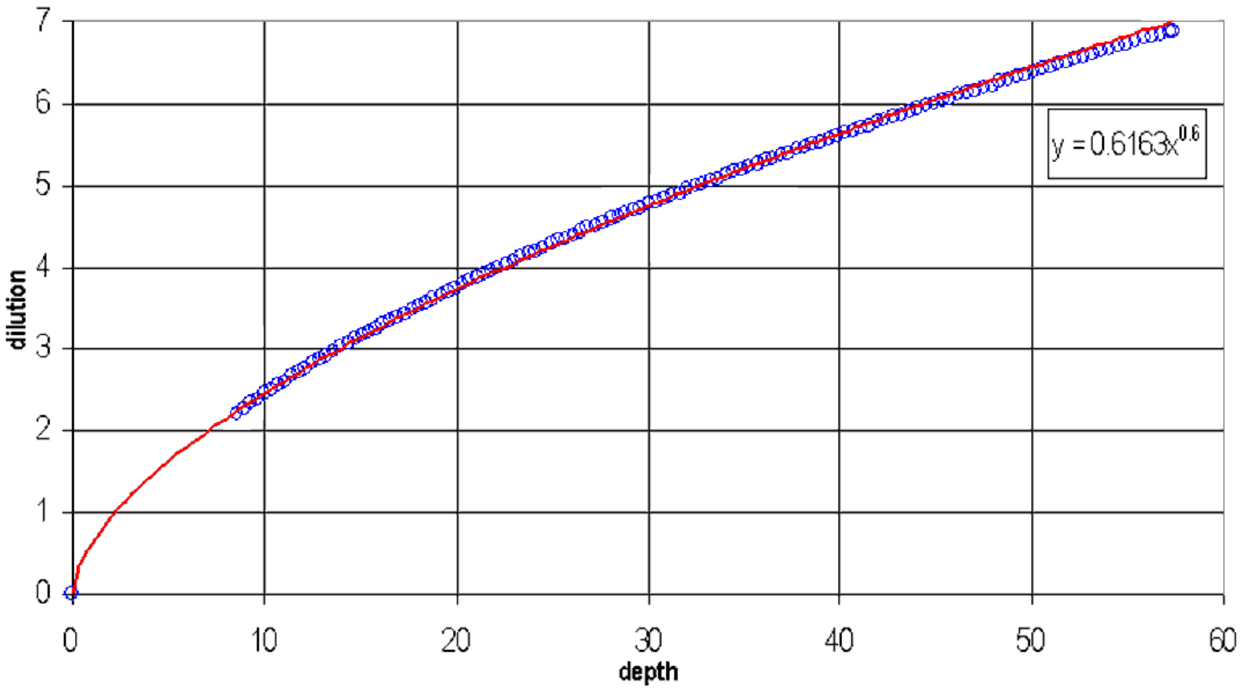

Figure 2. Sequoyah 2D Plume Model

The current one-dimensional plume model uses a separate entrainment coefficient for buoyancy-driven and momentum-driven entrainment. The effective entrainment coefficient for a momentum-dominated plume based on the zero-dimensional TVA plume model is compared to the one-dimensional ZZ plume model in the following figure.

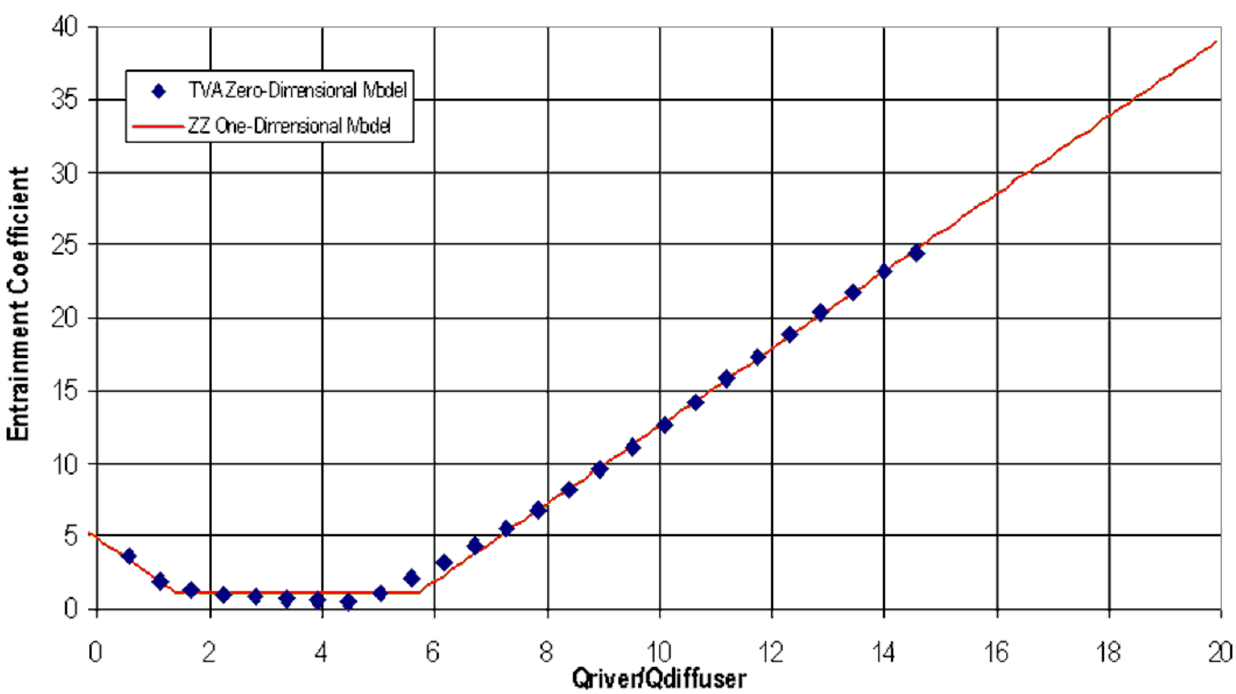

Figure 3. Velocity Entrainment Coefficient

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