Development of a Two-Dimensional Plume Model For Positively and Negatively Buoyant Discharges Into a Stratified Flowing Ambient

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The development of a computer model for a two-dimensional round or slot jet plume is presented. The plume can be positively, negatively, or neutrally buoyant with respect to the ambient, and composed of heated or cooled fresh or salty water. The ambient can be stratified or uniform in temperature, flowing, stagnant, fresh or salty. Samples of computed trajectory and dilution are also presented as well as comparisons with field data.

Introduction

Thermal and wastewater effluents are frequently discharged into a receiving ambient via a diffuser. The primary purpose of this is to dilute the effluent. The most common diffuser shapes are assumed to be similar to a solitary round jet or a linear cluster of jets which approach an ideal slot. The discharge is most often hotter or equal to the ambient in temperature, but could be colder. Furthermore, the discharge and the ambient can contain varying amounts of salt. The discharge can either rise, fall, spread, or some combination of these. The trajectory as well as the final dilution is of interest to the engineer.

Most often such discharges are turbulent yet quasi-steady on a gross scale, although not on a local scale. A plume model is one alternative between developing an empirical correlation for the dilution and destination of the discharge and comprehensive three-dimensional numerical modeling of the locally unsteady turbulent mixing phenomena. The former requires a number of laboratory and prototype tests which can be very costly; and the latter involves substantial computational expense. Various plume models have been developed. The distinctiveness of the present plume model are its simplicity and avoidance of classical errors.

Model Assumptions

In order to model a discharge as a plume, there are several assumptions which are made:

1. The plume is assumed to be a coherent structure which may pass through the ambient or the ambient through it, yet still retain properties distinct from the ambient.

2. The plume although subjected to small and large scale turbulence is assumed to be quasi-steady on a gross scale.

3. The plume is assumed to be a perturbation when compared to the ambient.

4. The properties of the plume are assumed to be characterized by average measures.

5. It is assumed that the extent of the plume into the ambient and its distinctiveness from the ambient can be delineated by a boundary.

6. The interaction between the plume and the ambient is assumed to occur at the boundary and be limited to entrainment.

Although these assumptions may seem to be rather sweeping, it will be shown subsequently that meaningful, and even surprisingly accurate, results can be obtained from such a model. The principle features of the plume and ambient are shown in Figure 1.

Thermophysical Properties

The density and specific heat of a brine depend on the temperature and salinity. These relationships can be expressed by the following series:

$$\rho(T,S)=\sum\sum A_{I,J}\left(\frac{T}{100}\right)^{I}S^{J} \tag{1}$$

$$C_p(T,S)=\sum\sum B_{I,J}\left(\frac{T}{100}\right)^{I}S^{J} \tag{2}$$

Equations 1 and 2 are normal bi-variate polynomials. These were determined from a least-squares curve-fit of ASHRAE tabulated data for NaCl brine. I addition to the least-squares fit, "knots" (constrained points of exact agreement) were "tied" at the points of maximum density and unity specific heat. The data cover a range of temperatures from 0 to 100°C and a salinity of 0 to 0.26 (or 26 percent by weight). The density is in grams per cubic centimeter, and the specific heat is in calories per gram per degree C. the average and maximum errors with respect to tabulated data are 3 and 9 parts per 100,000 respectively for the density of fresh water (S=0) and 7 to 24 parts per 100,000 respectively for salinities up to 0.26. The corresponding errors for the specific heat are 33, 118, 44, 177 parts per 100,000.

Entrainment

The entrainment velocity, G, has been computed in many ways by various investigators. Numerous relationships exist in the literature. A simple and satisfactory relationship based on the densimetric Froude number, Fr, is given by Fischer, et al.

$$G_{slot}=0.0520\ \left(|U_p-U_a|+|V_p-V_a|\right)\ e^{\left(\frac{1.62}{Fr^{1.5}}\right)}$$
$$\text{where}\quad Fr=\frac{W}{\sqrt{gb\frac{|\rho_a-\rho_p|}{\rho_a}}} \tag{3}$$

$$G_{round} = 0.0535\ \left(|U_p - U_a| + |V_p - V_a|\right)\ e^{\left(\frac{1.43}{Fr^2}\right)}$$
$$\text{where} \quad Fr = \frac{W}{\sqrt{gR\frac{|\rho_a - \rho_p|}{\rho_a}}} \qquad \textbf{(4)}$$

where U_p and U_a are the horizontal velocities of the plume and ambient respectively, Vp and Va are the vertical velocities of the plume and ambient respectively, and ρ_p and ρ_a are the densities of the plume and ambient respectively. R is the radius, b is the width of the plume, and g is the acceleration of gravity. Equation 3 is used with slot jets and Equation 4 is used with round jets.

While the entrainment at the upper and lower or upstream and downstream boundaries of the plume are no doubt different, there is insufficient experimental information to separate the two. Furthermore, separating the two would be inconsistent with the assumed coherent structure, quasi-steady behavior, and perturbatory effects; or "gnat-straining" when compared to the other assumptions.

Internal Profiles

It is common in plume analysis to assume that the internal density, salinity, and velocity take on some shape such as a Gaussian distribution with a maximum at the centerline and blending into the ambient at the boundary. This certainly seems like a reasonable assumption and what one would expect to see from a three-dimensional numerical model. Finding such a profile in the field downstream of a large thermal plume, at least, is quite another matter (McIntosh, Johnson, Speaks, and Ungate, and Howerton).

Furthermore, problems arise with internal profiles when considering entrainment. If the entrainment is assumed to be proportional to the difference between the ambient velocity and the maximum velocity within the plume, then the average velocity within the plume will never equal the ambient velocity, regardless of the duration of interaction. If the entrainment is assumed to be proportional to the difference between the ambient velocity and the average velocity within the plume, then the eventual average velocity of the plume will equal the ambient. However, the maximum velocity within the plume will exceed the ambient velocity. This will occur even if the ambient must accelerate the plume. This would imply that the ambient could increase the speed of the plume beyond its own; which, of course, is impossible.

Assuming an internal profile within a plume is either an attempt to add extra information to or extract additional information from a model on a level of detail exceeding the limitations inherent in the assumptions, the rewards of which are doubtful.

Governing Equations

The governing equations which are applied to this plume are: the conservation of mass, energy, and linear momentum. When there is salt in either the discharge or ambient, it is necessary to have two conservation of mass equations (one for the water and one for the salt). These are Equations 5 and 6 respectively (refer to Figure 1 for details).

$$\frac{d(\rho_p Wb)}{dZ}=\rho_a G \tag{5}$$

$$\frac{d(S_p \rho_p Wb)}{dZ}=S_a \rho_a G \tag{6}$$

where W is the average velocity of the plume along the centerline, S_p and S_a are the salinity of the plume and ambient respectively, and Z is the centerline coordinate. The conservation of energy is applied similarly.

$$\frac{d(E_p \rho_p Wb)}{dZ}=E_a \rho_a G \tag{7}$$

where E_p and E_a are the energy per unit mass of the plume and ambient respectively. The conservation of linear momentum is applied in two orthogonal directions: horizontal and vertical (indicated by X and Y respectively in Figure 1).

$$\frac{d(U_p \rho_p Wb)}{dZ}=U_a \rho_a G \tag{8}$$

$$\frac{d(V_p \rho_p Wb)}{dZ}=V_a \rho_a G+(\rho_a-\rho_p)gb \tag{9}$$

Equations 5 through 9 apply to a slot jet. For a round jet these become:

$$\frac{d(\rho_p W\pi R^2)}{dZ}=\rho_a G2\pi R \tag{10}$$

$$\frac{d(S_p \rho_p W\pi R^2)}{dZ}=S_a \rho_a G2\pi R \tag{11}$$

$$\frac{d(E_p \rho_p Wb\pi R^2)}{dZ}=E_a \rho_a G2\pi R \tag{12}$$

$$\frac{d(U_p \rho_p W\pi R^2)}{dZ}=U_a \rho_a G2\pi R \tag{13}$$

$$\frac{d(V_p \rho_p W\pi R^2)}{dZ}=V_a \rho_a G2\pi R+(\rho_a-\rho_p)g\pi R^2 \tag{14}$$

The trajectory of the plume (X_p,Y_p) is computed from Equations 15 and 16.

$$\frac{dX_p}{dZ}=U_p \tag{15}$$

$$\frac{dY_p}{dZ}=V_p \tag{16}$$

Solution of the Governing Equations

These seven ordinary differential equations (either Equations 5-9, 15, 16 for a slot jet or 10-16 for a round jet) describe the behavior of the plume. In addition to these differential equations, it is necessary to prescribe the initial conditions and the conditions of the ambient. Once these are known, the trajectory and dilution of the plume can be determined by solving the equations numerically. A satisfactory method is 4th-Order Runge-Kutta. A limited geometric progression of the step size can be used to speed the process without loss of accuracy.

In order to solve the equations it is necessary to obtain through an iterative process the secondary parameters. T_p, S_p, and ρ_p as these are implicit along with the primary parameters U_p and V_p. One algorithm for accomplishing this is to assume the previous values for T_p and S_p. This will then permit the calculation of ρ_p and E_p. Dividing the integrands of Equations 6 by 5 and 7 by 5 yields S_p and E_p respectively. These will not typically agree with the provisional value of S_p and the computed value of E_p based on the provisional values of T_p and S_p. The provisional values of S_p and T_p must then be corrected until there is acceptable agreement. Successive substitution of S_p and T_p has proven satisfactory (noting that T_p is essentially proportional to E_p as the specific heat varies only slightly).

Once this iterative process is completed, the other plume parameters can be obtained. Dividing the integrands of Equations 8 by 5 and 9 by 5 yields U_p and V_p respectively. The trajectory follows immediately from Equations 15 and 16.

Common Pitfalls and Misconceptions

First, it is always good practice to solve differential equations in conservative form. In this case the primary integrand should be the quantity being conserved. Specifically, mass, salt, energy, horizontal linear momentum, and vertical linear momentum respectively. While it is possible to formulate differential equations in terms of the secondary or non-conserved quantities such as velocity, salinity, and temperature, it is neither necessary nor advantageous. Solution of the conservative form assures conformance with the governing principles; whereas, the other does not.

Second, the conservation principles should be imposed rather than a special case or subset. Three common illustrations are the "conservation" of buoyancy, volume, and temperature. Tatom explains the "conservation" of buoyancy:

> The concept of the conservation of buoyancy appears to have been approximately [34] years ago in the analysis of buoyant gas plumes in the atmosphere (Morton, Taylor, and Turner). As indicated in that analysis, this conservation is actually an approximate form of the conservation of energy equation, but for gases at atmospheric pressure it is quite sufficient... the conservation of buoyancy principle has been applied to buoyant plumes i water... In this situation, however, the derivation based on conservation of energy is quite tenuous... the effect on such an application is a basic loss of accuracy.

The "conservation" of volume is but a special case of the conservation of mass when density is constant; while the "conservation" of temperature is but a special case of the conservation of energy when both the density and specific heat are constant. Neither simplification can be appropriately applied to plumes. Furthermore, it is simply not necessary; and, in fact, complicates the equations when numerical methods are available. As Tatom points out, there is some simplification derived from the "conservation" of buoyancy when limited to analytical means; however, the advent of digital computers eliminates any such advantage.

The previous remarks are from a theoretical perspective. From a practical perspective, comparisons performed by the author and Frank Tatom of several models based on the conservation formulation and the "conservation" of buoyancy, volume, and temperature revealed moderate differences for thermal plumes in water except under extreme temperature gradients. Tatom gives several figures and tables for various cases. It could certainly be argued that these errors introduced in the formulation are minor in comparison to those introduced by the basic plume assumptions. However, these are unnecessary and provide no simplification.

Comparison with Laboratory and Field Data

Figure 2 shows the upstream temperature profile in a moderately deep reservoir (15 meters) under nearly uniform conditions at six-hour intervals over a period of four days. Figure 3 shows the upstream profiles five months later under strongly stratified conditions. Figure 4 shows the temperatures at the same location two days later after mixing occurred in the upper layer. Figures 5 and 6 show the river flows during the same periods. The difference between the river temperature at the 1.5 meter depth downstream of the diffuser and upstream as measured and computed by the plume model for these periods is shown in Figure 7 through 9. The 1.5 meter depth is compared as this is the point of compliance with thermal water quality standards, and thus the focal point of field studies and continuous monitoring. The calculations were performed at one-hour intervals using 15-minute running average data and assuming steady-state behavior during each interval.

The agreement between the plume model and measurements over this period when the upstream stratification was changing, as was the river flow and discharge temperature is quite good. It is highly unlikely that such close "tracking" could arise from a synergistic combination of mutually cancelling errors.

Figure 10 shows the agreement between the plume model and measurements for a range of dilutions. Figure 10 shows both laboratory and prototype data. The difference in scale between the laboratory and prototype was 90:1. These data are from McIntosh, et al. and Ungate et al.

Sample Trajectories Asymptotic Behavior

Figures 11 and 12 show the computed trajectory of the plume centerline for a range of ambient river flows. Figure 11 is for a positively buoyant thermal plume and Figure 12 is for a negatively buoyant salty plume.

Figures 13 through 15 show the asymptotic behavior of the computed dilution of the plume for Froude numbers of 1, 10, and 100 respectively. Here a thermal plume is discharged upward into an infinite, uniform, stagnant ambient. Also illustrated in these figures are the results of three analytical methods (Almquist, Cedarwall, and Roberts). The plume results agree well with Cedarwall and lie between the results of Almquist and Roberts.

Summary

The development of a two-dimensional plume model for a round or slot jet has been presented. The formulation of this model is based on the conservative form of the governing equations. The model can handle combinations of heated and salty discharges and flowing ambients. The model results compare favorably with laboratory and field data. When confined to an ideal thermal plume, the asymptotic behavior of the model agrees well with an analytical method.

References

Almquist, C. W., "Submerged Multiport Diffuser Analysis and Design for Hartsville Nuclear Plant," TVA Report No., WR28-1-89-100, September 1978.

Cedarwall, K., "Gross Parameter Solutions of Jets and Plumes," Journal of the Hydraulics Division, ASCE, 101:HY5, May 1975.

Fischer, H. B., J. Imberger, E. J. List, R. C. Y. Koh, and N. H. Brooks, Mixing in Inland and Coastal Waters, Academic Press, New York, 1979.

McIntosh, D. A., B. E. Johnson, and E. B. Speaks, "A Field Verification of Sequoyah Nuclear Plant Diffuser Performance Model: One-Unit Operation," TVA Report No. WR28-1-45-110, October 1982.

McIntosh, D. A., B. E. Johnson, and E. B. Speaks, "Validation of Computerized Thermal Compliance and Plume Development at Sequoyah Nuclear Plant," TVA Report No. WR28-1-45-115, August 1983.

Roberts, P. J. W., "Line Plume and Ocean Outfall Dispersion," Journal of the Hydraulics Division, ASCE 105:HY4, April 1979.

Tatom, F. B., "Errors Associated with the Use of the Conservation of Buoyancy Concept," Engineering Analysis Inc., Report No. EAI-TM-83-02A, December 1983.

Ungate, C. D., and K. A. Howerton, "Effect of Sequoyah Nuclear Plant Discharges on Chickamauga Lake Water Temperatures," TVA Report No. WR28-1-45-101, April 1978, revised March 1979.

Nomenclature

Symbol	Meaning
b	plume width perpendicular to the centerline
E	specific energy
Fr	Froude Number
g	gravitational acceleration
G	entrainment velocity
ρ	density
R	plume radius perpendicular to the centerline
S	salinity (salt fraction by weight)
T	temperature
U	horizontal velocity
V	vertical velocity
W	magnitude of the velocity
X	horizontal coordinate
Y	vertical coordinate
Z	centerline coordinate

Subscripts

a	ambient
p	plume

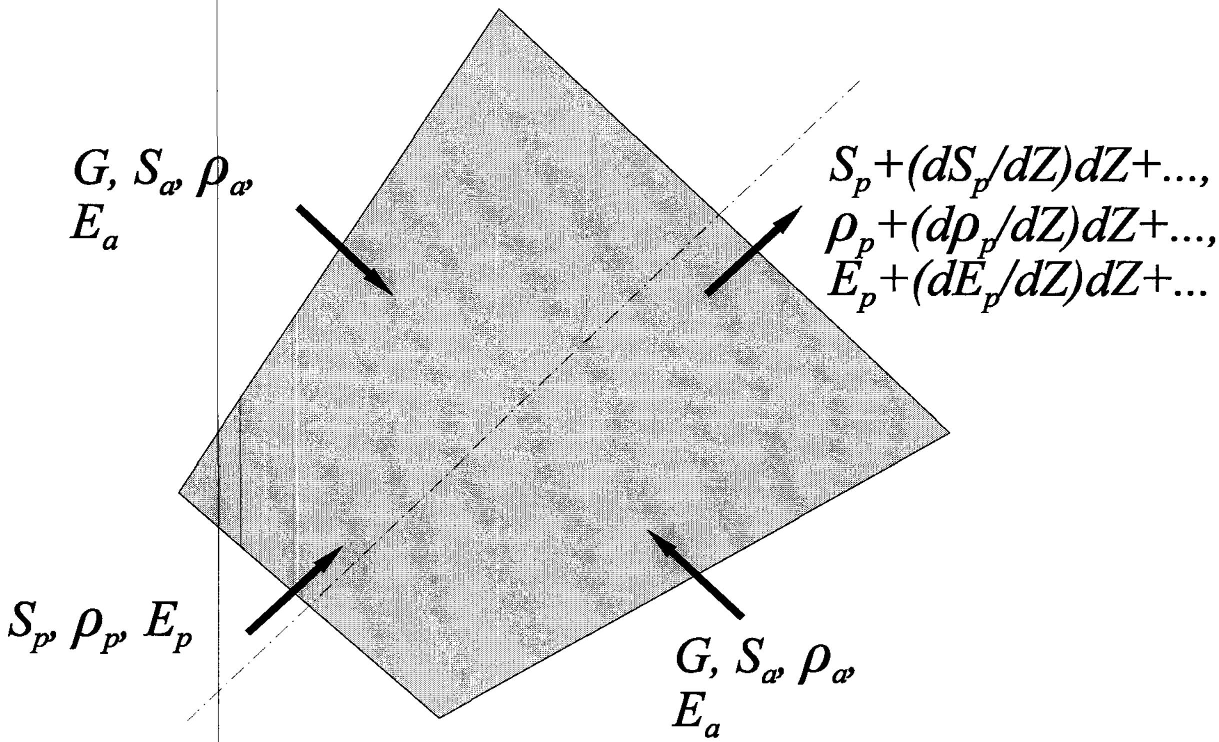

Figure 1. Pume Element

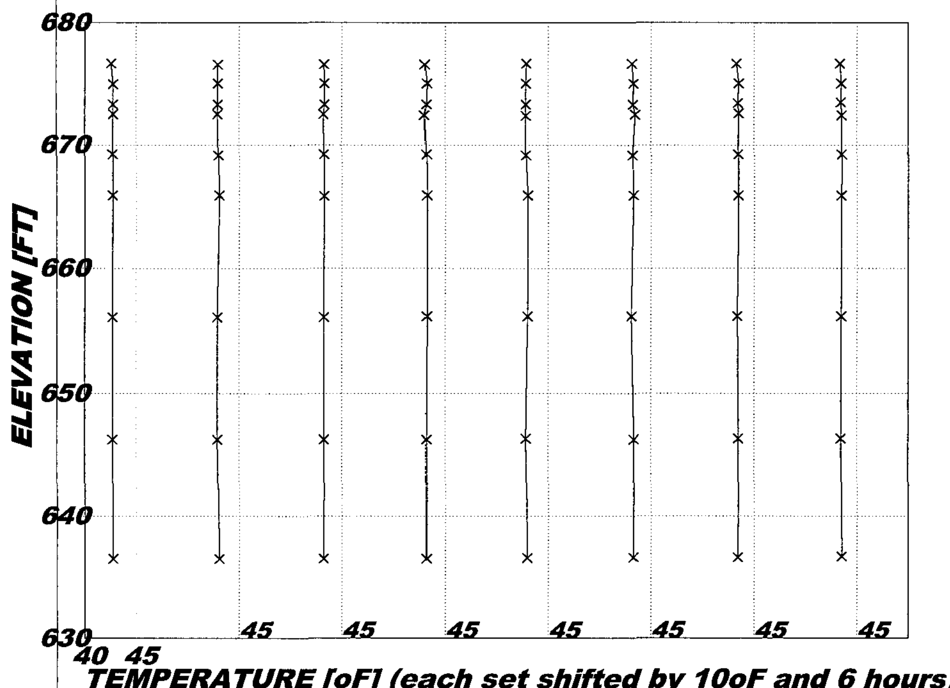

Figure 2. Upstream Temperature Profiles (December 28&29, 1980)

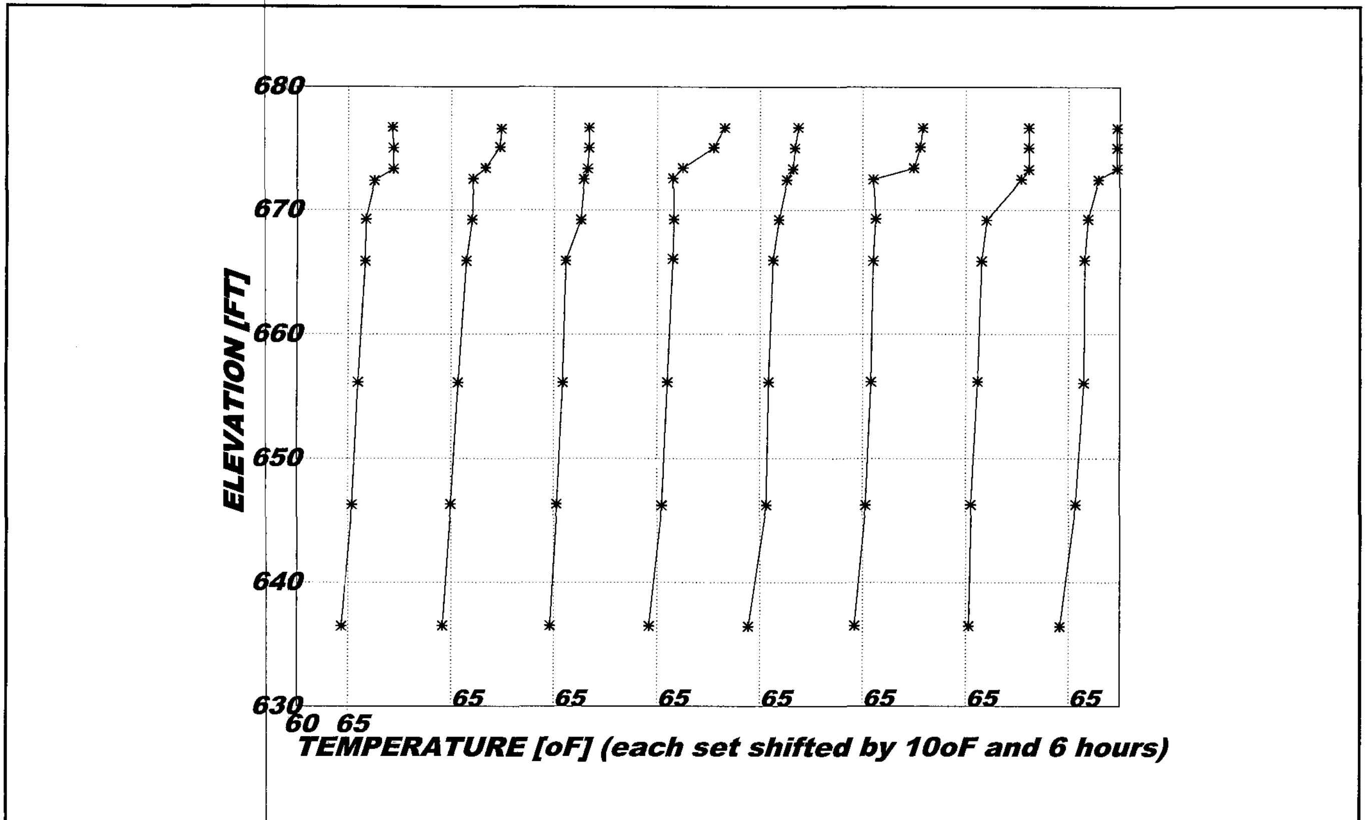

Figure 3. Upstream Temperature Profiles (May 5&6, 1981)

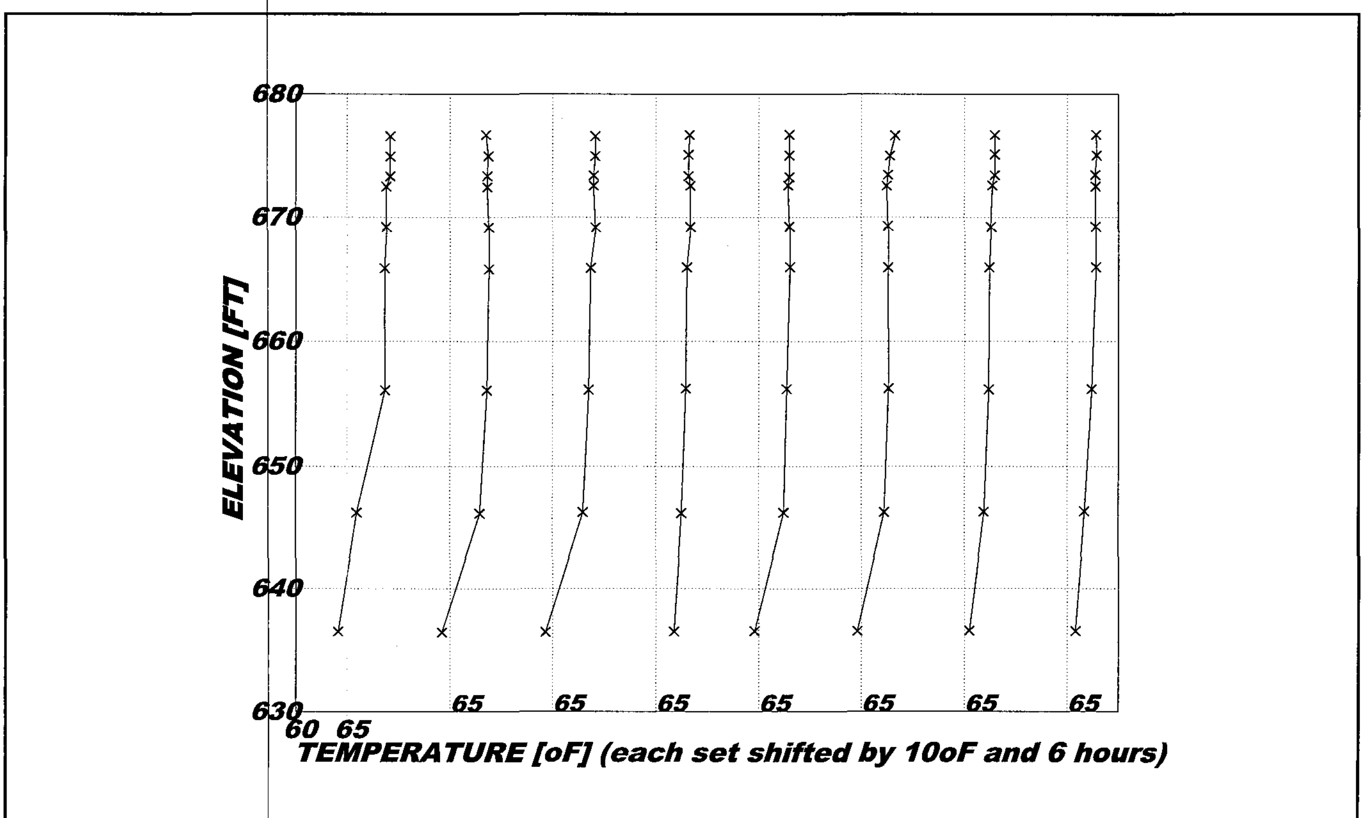

Figure 4. Upstream Temperature Profiles (May 7&8, 1981)

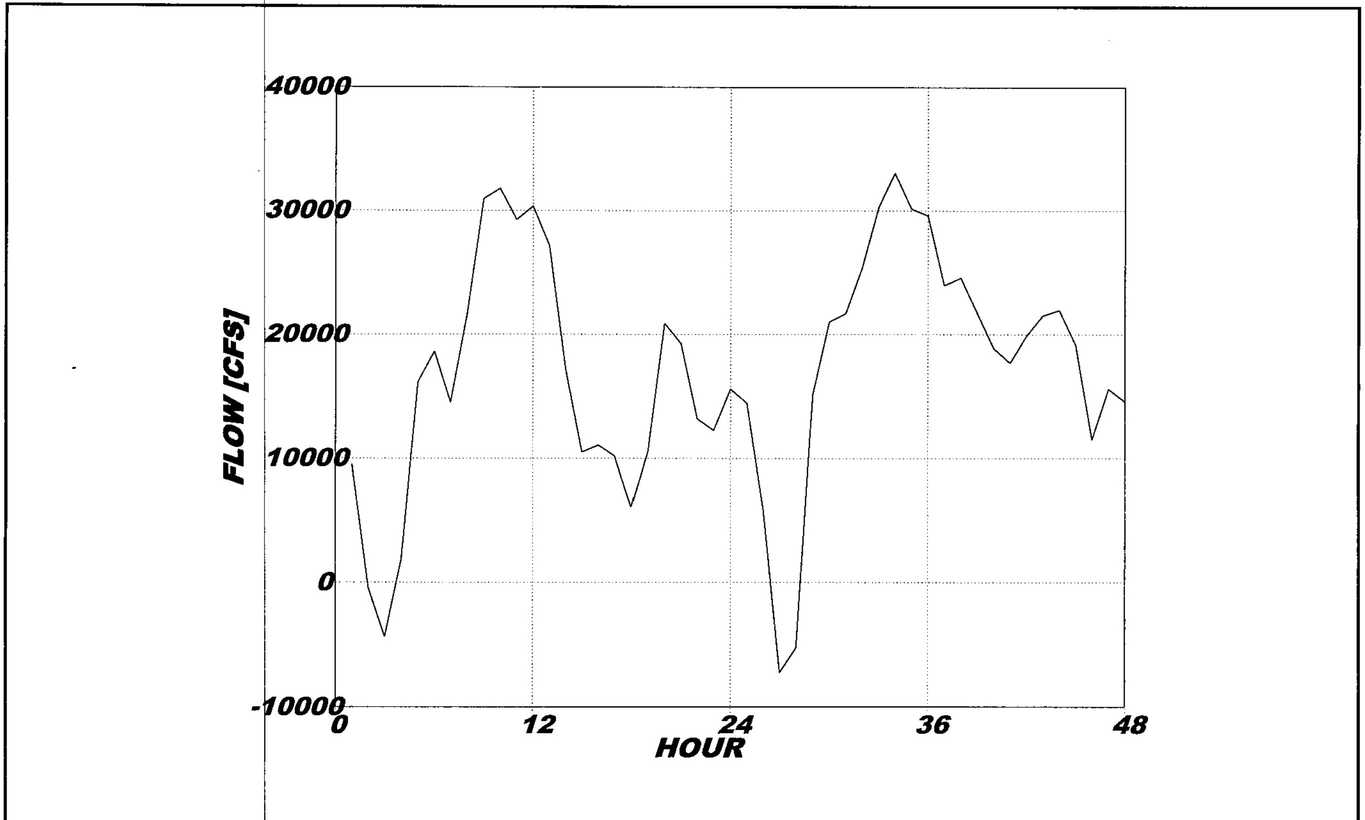

Figure 5. River Flow (December 28&29, 1980)

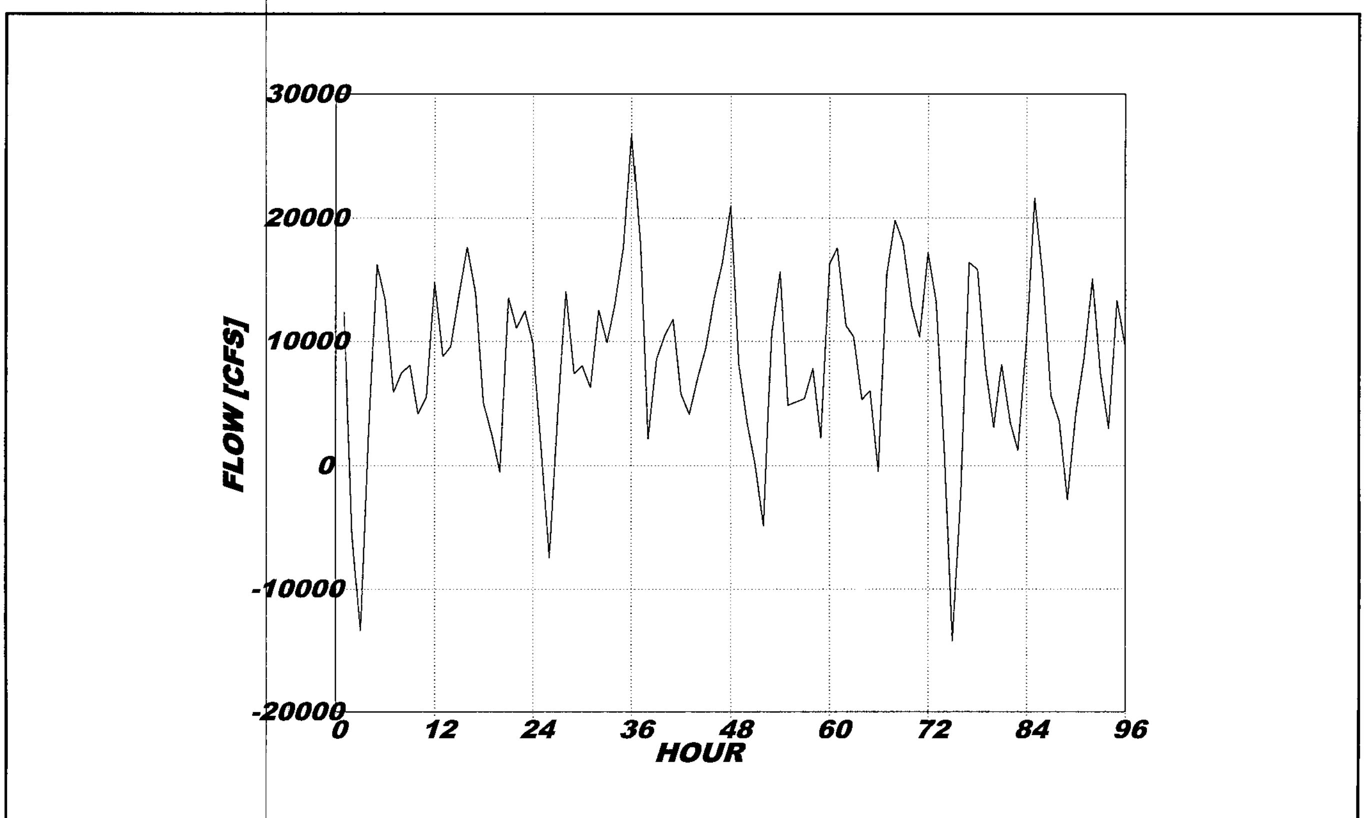

Figure 6. River Flow (May 5-8, 1981)

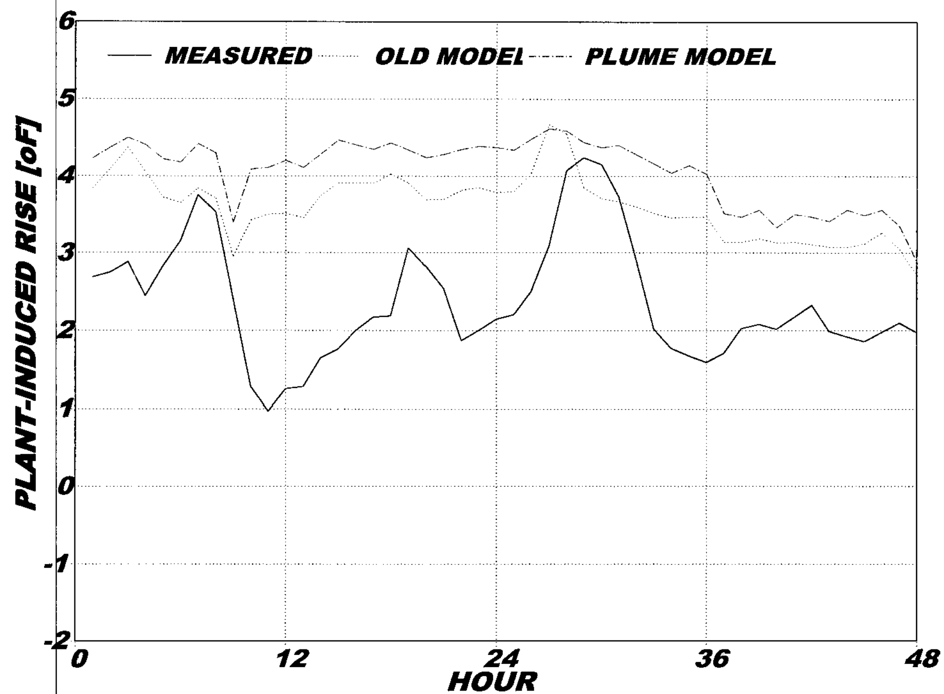

Figure 7. Plant-Induced Rise (December 28&29, 1980)

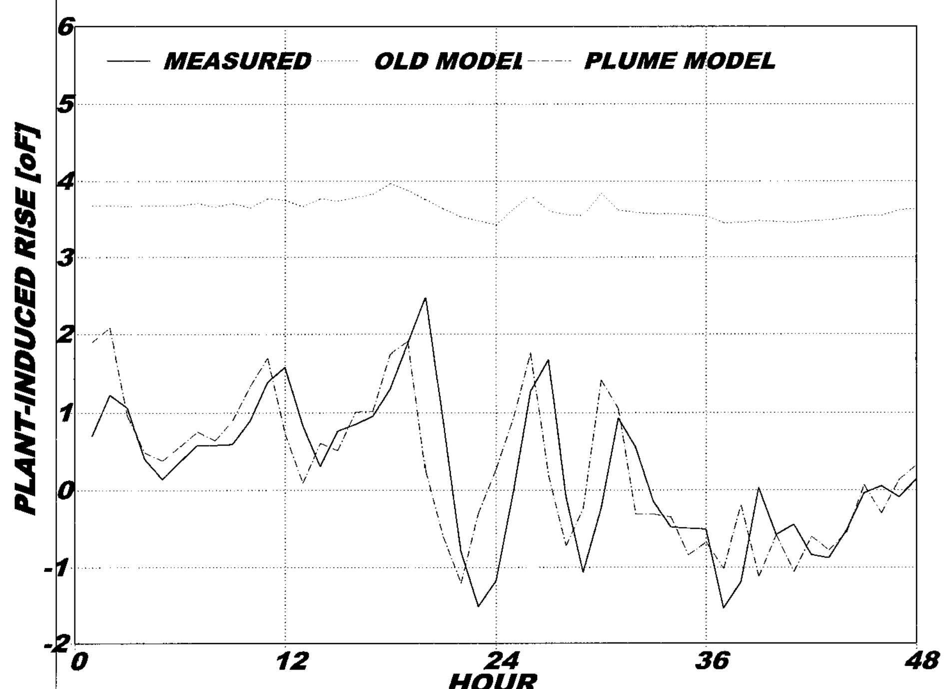

Figure 8. Plant-Induced Rise (May 5-8, 1981)

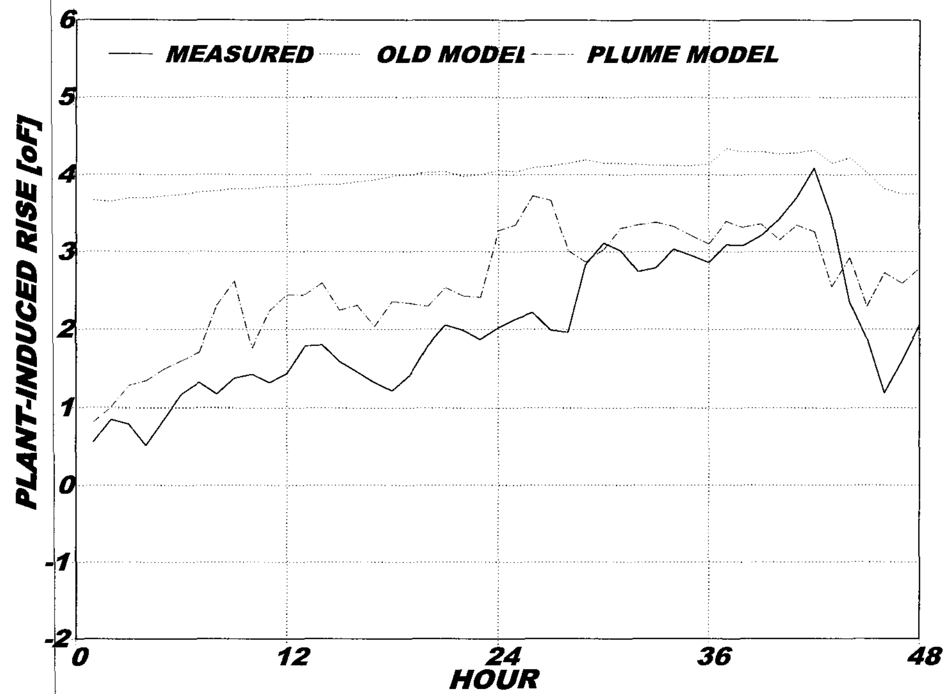

Figure 9. Plant-Induced Rise (May 7&8, 1981)

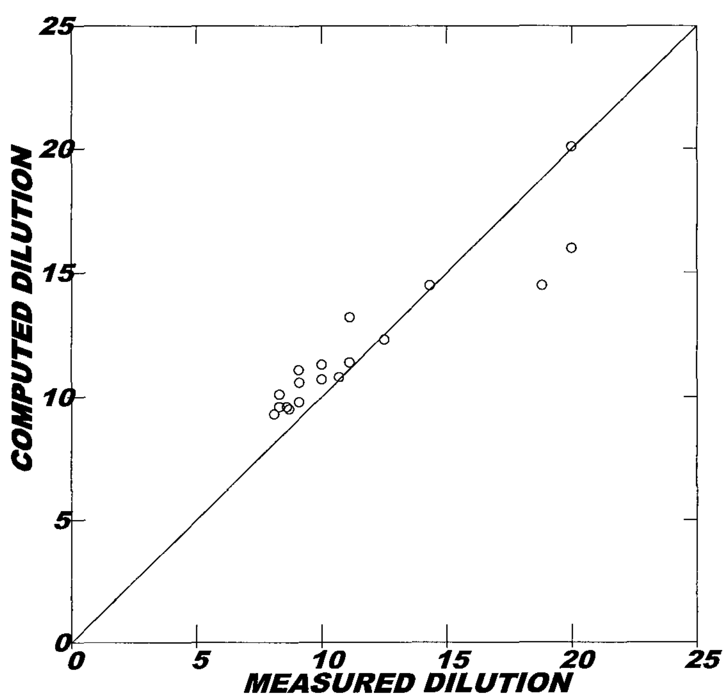

Figure 10a. Comparison of Plume Model and Lab Data

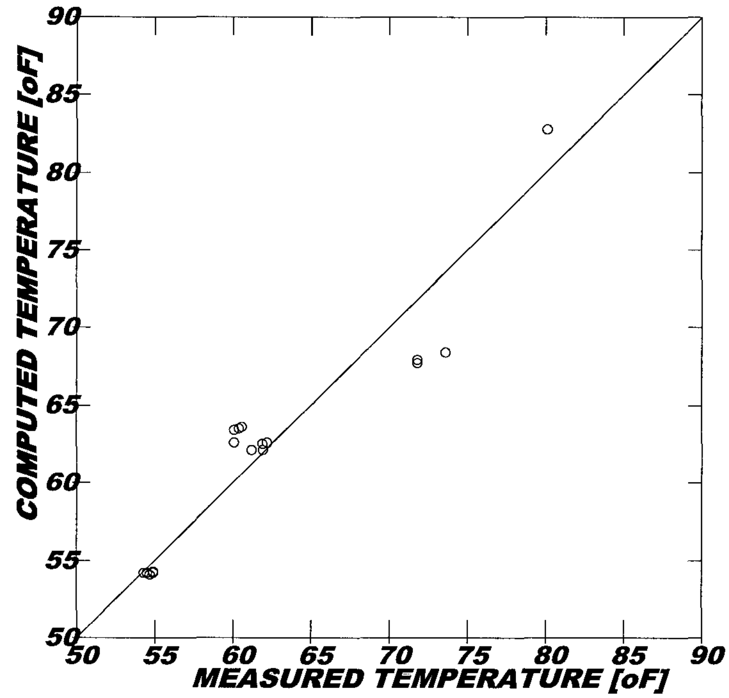

Figure 10b. Comparison of Plume Model and Field Data

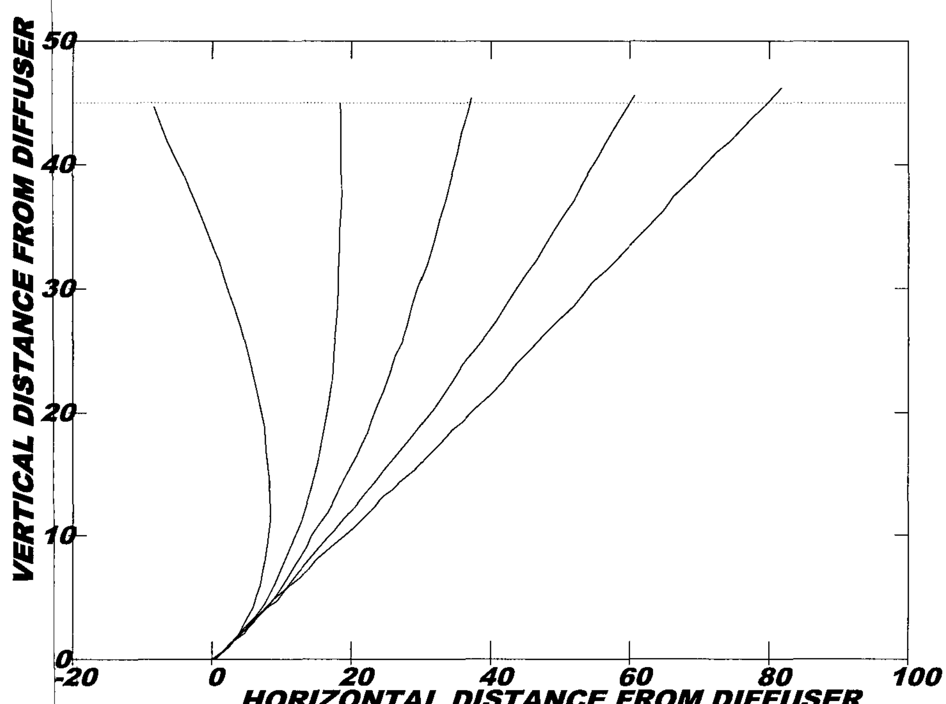

Figure 11. Computed Thermal Plume Trajectories

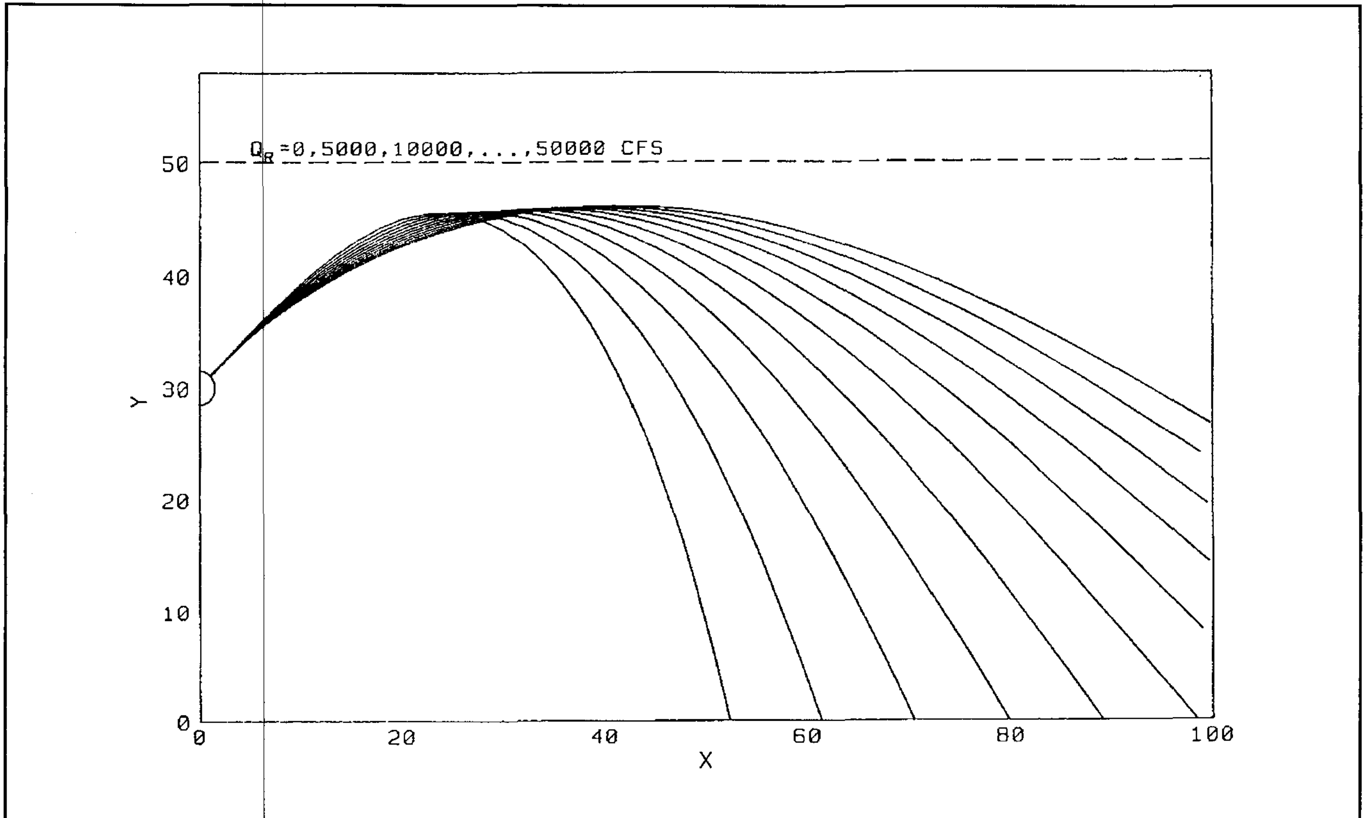

Figure 12. Computed Saline Plume Trajectories

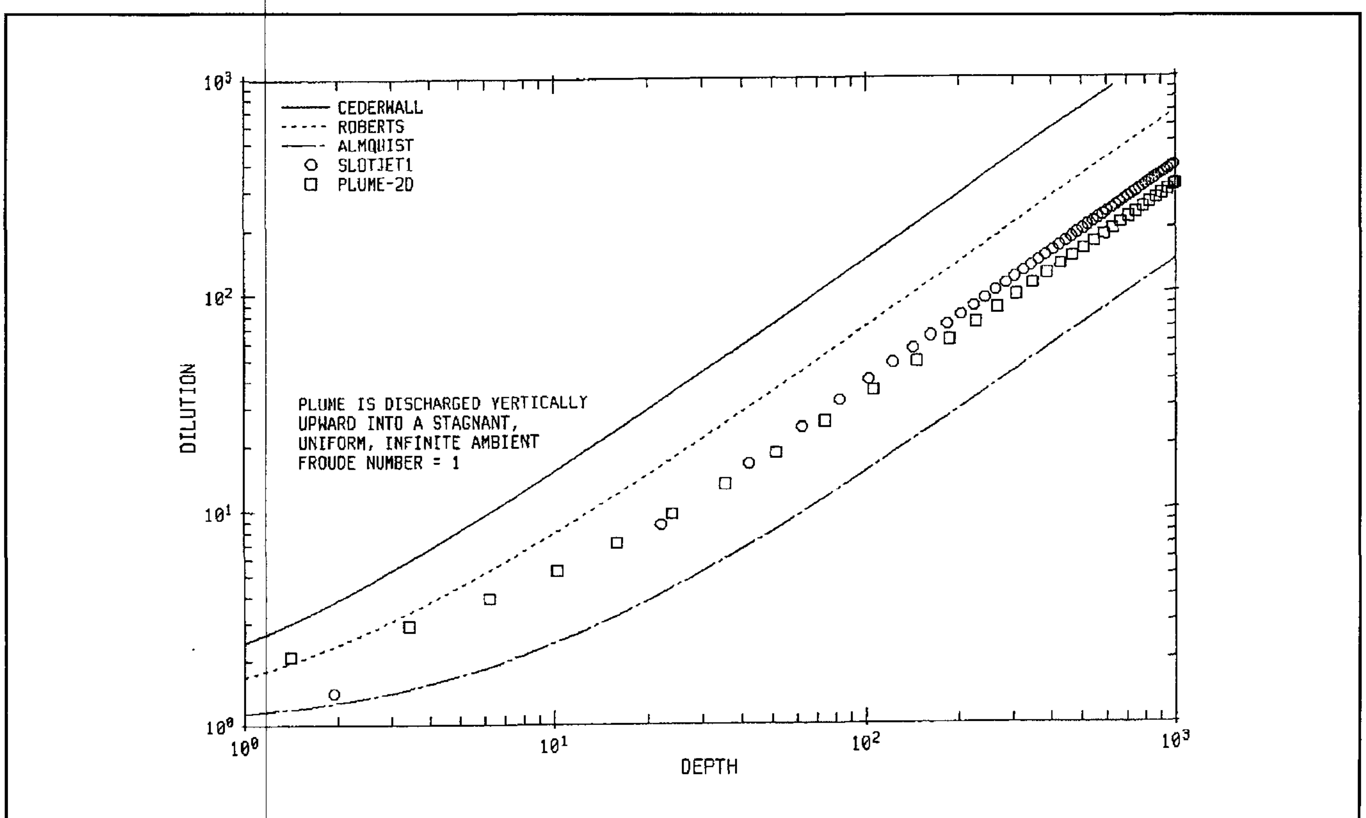

Figure 13. Comparison of Plume and Analytical Models (Froude Number = 1)

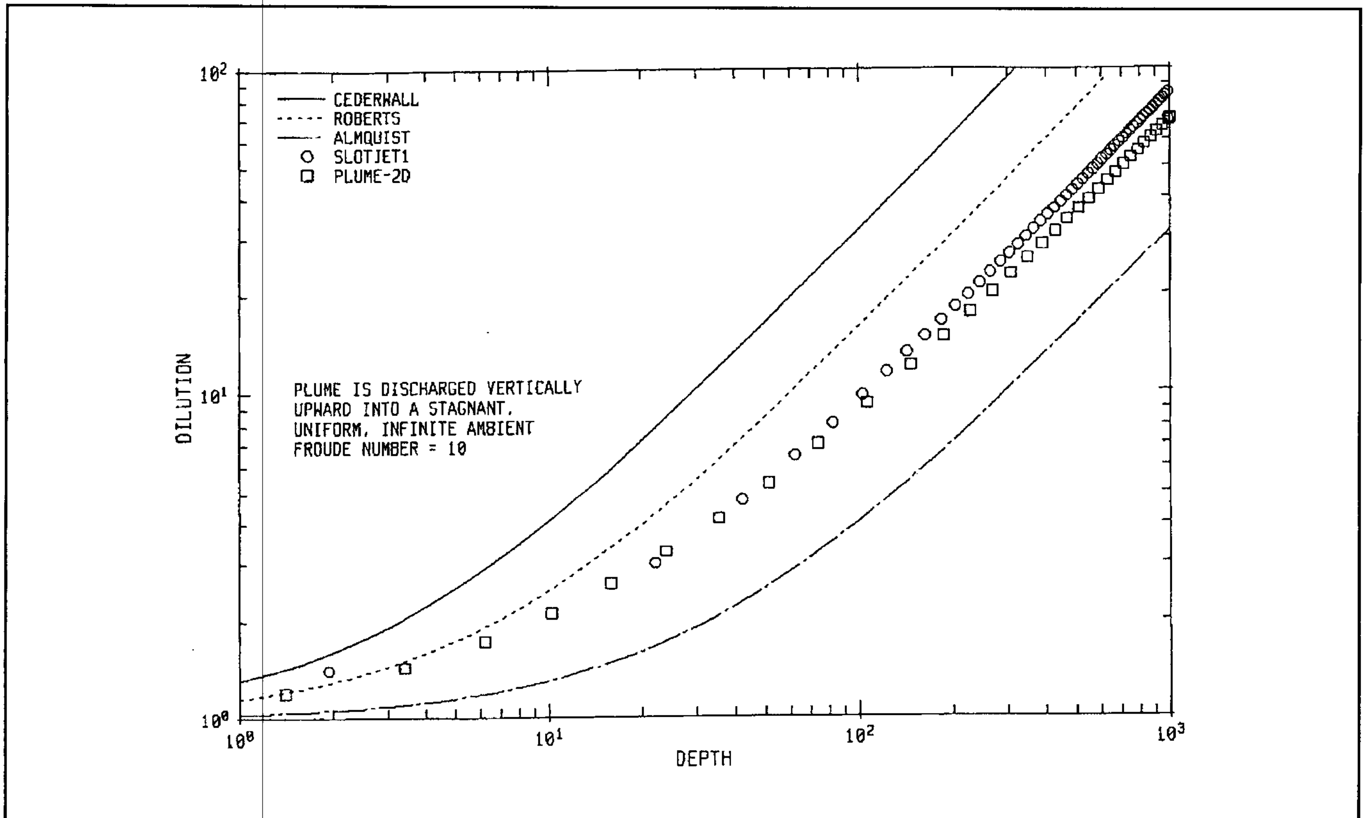

Figure 14. Comparison of Plume and Analytical Models (Froude Number = 10)

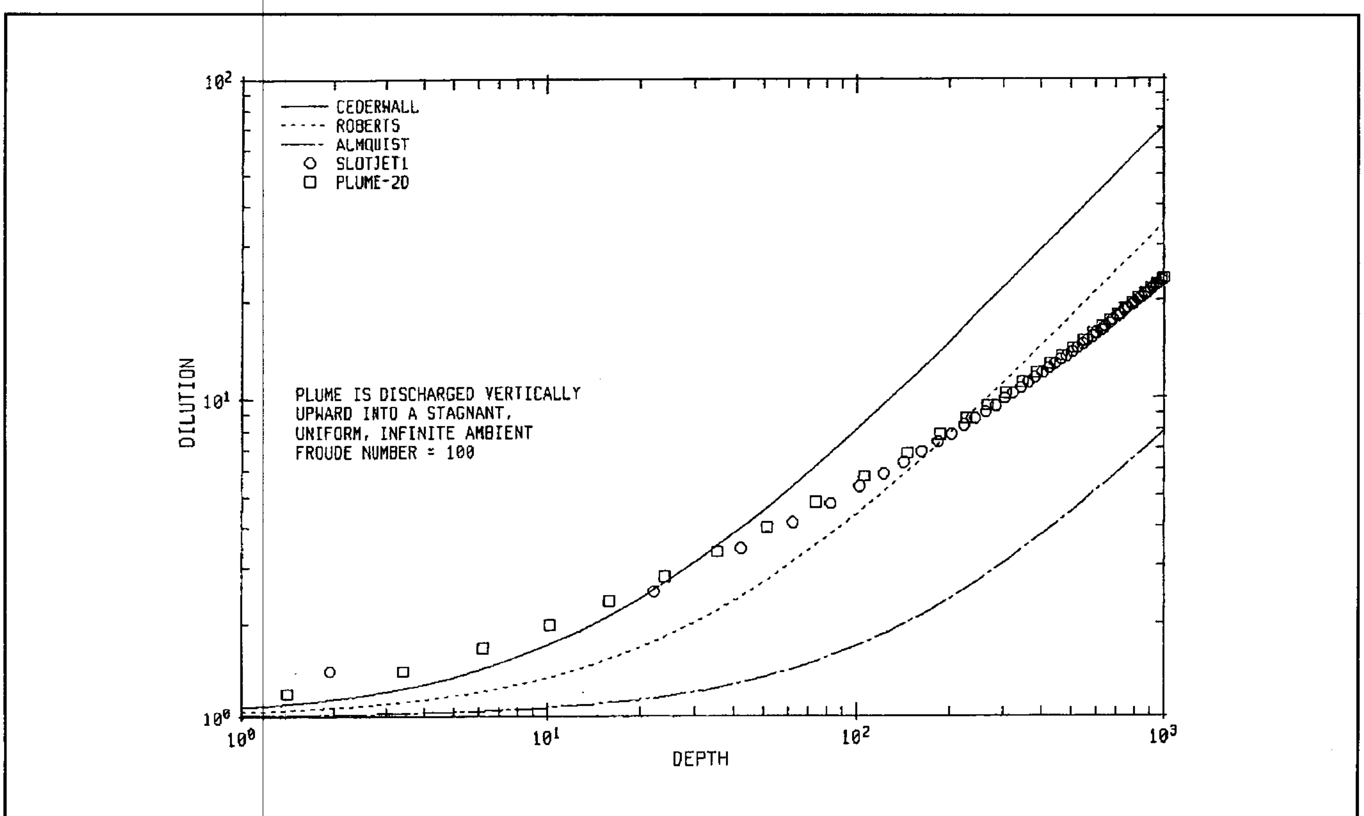

Figure 15. Comparison of Plume and Analytical Models (Froude Number = 100)