presented at the
Fifth Cooling Tower Workshop of the
International Association for Hydraulic Research
Palo Alto, California
September 29 - October 3, 1986

MASS TRANSFER AND PRESSURE DROP IN SPRAYS FALLING IN A FREESTREAM AT VARIOUS ANGLES

by Dudley J. Benton and Robert L. Rehberg
TVA Engineering Laboratory

ABSTRACT

A theoretical analysis of mass transfer and pressure drop for a spray is presented and compared to experimental data. The spray consists of 1-mm to 5-mm diameter water droplets falling in air that is flowing at an angle of between 90 and 180 degrees relative to gravity. The analysis is particularly applicable to the rain zone of natural draft counterflow cooling towers where the angle between the falling drops and the air varies. The results of the analysis are presented in a form that is familiar to the cooling tower industry.

INTRODUCTION

The thermal efficiency of many power plants is directly related to the efficiency of the plants' cooling towers which are used in the recirculation loop of the condenser cooling water. Several computer codes, such as the TVA FACTS (Fast Analysis Cooling Tower Simulator) [1] code, have been developed to model the heat transfer and pressure drop phenomena in a cooling tower. Measurements taken recently by the Environmental Systems Corporation [2] indicate that a significant percentage of the total heat transfer in a counterflow natural draft cooling tower occurs in the rain zone below the fill. Therefore, an increased understanding of the heat transfer and pressure drop occurring in the rain zone is essential to the enhancement of modeling techniques. Natural draft counterflow cooling towers generally have base diameters of 300 to 350 feet with rain zone heights of 25 to 35 feet. All of the air entering the tower travels through this rain zone. As the air flows toward the center of the tower, a portion of the air turns upward to enter the fill, thereby assuming a counterflow orientation to the falling water. However, air reaching the center of the tower before turning upward has passed through as much as 175 feet of rain zone in a crossflow orientation (see Figure 1). Therefore, a study of droplet heat transfer and pressure drop in the rain zone is necessary for both counterflow and crossflow orientations.

THEORETICAL DEVELOPMENT

In a classic reference, Lowe and Christie [3] discuss a method developed by Nottage and Boelter [4] for calculating the mass transfer characteristic and pressure drop of droplets falling in counterflow. The mass transfer characteristic, Ka/L", of the droplets is defined as

$$\frac{Ka}{L''} = \frac{C_{PL}}{C_{PG}} \frac{\kappa_G}{\kappa_L} \frac{\Psi}{s_G} \qquad \frac{1}{ft} \tag{1}$$

The "dynamical function", Ψ, defined and tabulated by Nottage and Boelter, is a function of droplet diameter and has units of $(time)^{-1}$. The relative velocity of the falling droplet, s_a, is the difference between the free falling or terminal velocity, s_f, and the air velocity through the rain zone, s_G. The terminal velocity is a function of droplet diameter and is tabulated in Reference 4. The air velocity through the tower is defined as

$$s_G = \frac{G''V_a}{3600} \qquad \frac{ft}{s} \tag{2}$$

The pressure drop, dP, across a layer of spray $d\ell$ feet deep is given by

$$dP = \left(\frac{g}{g_c}\right) \frac{L''}{3600 s_a} d\ell \qquad \frac{lbf}{ft^2} \tag{3}$$

The pressure drop expressed as velocity heads, dN, lost per foot is

$$\frac{dN}{d\ell} = \frac{2g_c}{s_G^2} \frac{dP}{d\ell} \qquad \frac{heads}{ft} \tag{4}$$

Substitution of Equation 3 into Equation 4 yields

$$\frac{1}{L''} \frac{dN}{d\ell} = \frac{2g}{3600\ \rho s_a s_G^2} \qquad \frac{ft\ hr}{lbm} \tag{5}$$

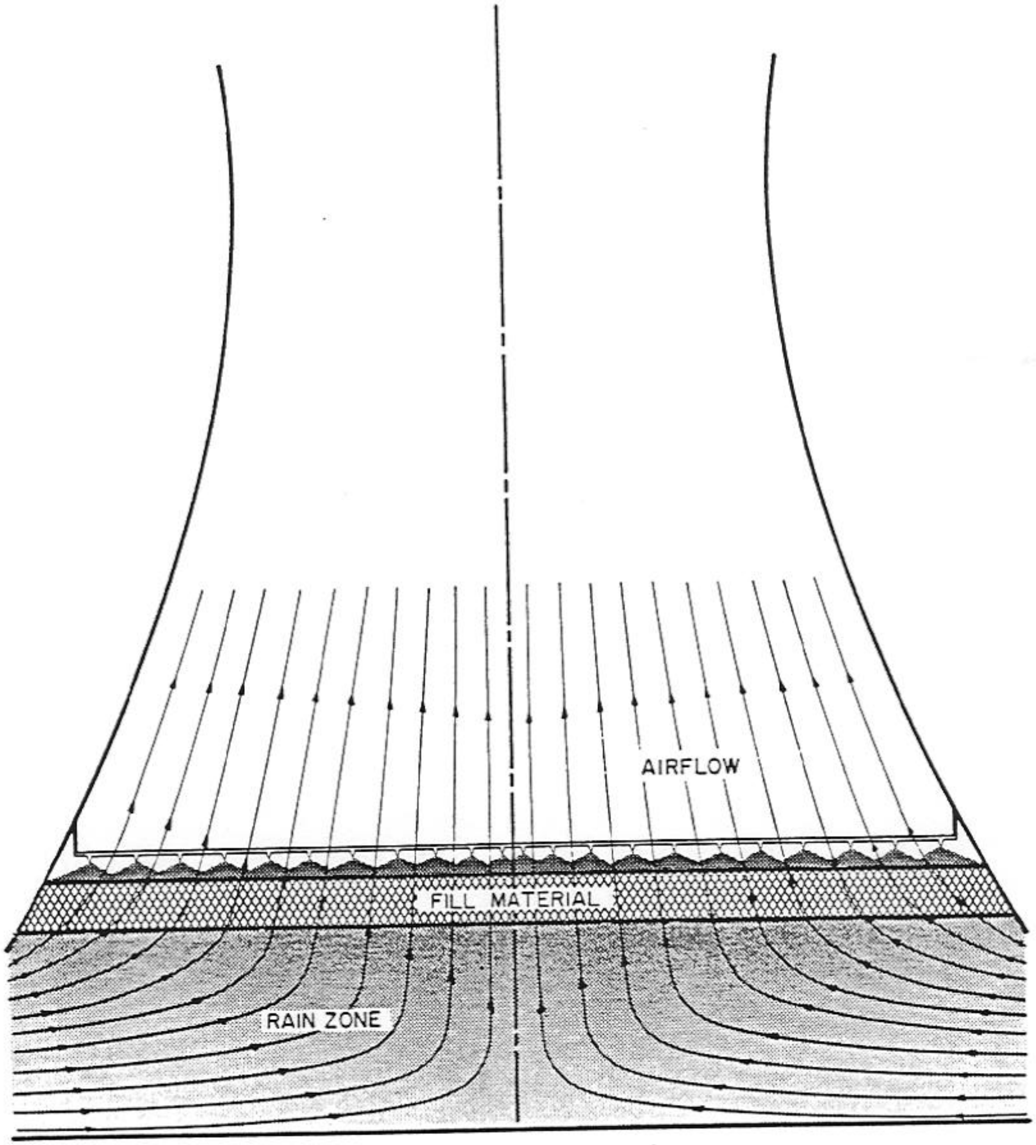


Figure 1. Orientation of Airflow in Natural Draft Counterflow Tower

For an example of the use of these equations in determining the mass transfer and pressure drop of falling droplets, assume the following physical properties:

$$C_{PL} = 1.00 \frac{\text{Btu}}{\text{lbm°F}}$$

$$C_{PG} = 0.24 \frac{\text{Btu}}{\text{lbm°F}}$$

$$\kappa_L = 0.35 \frac{\text{Btu}}{\text{hr ft°F}}$$

$$\kappa_G = 0.15 \frac{\text{Btu}}{\text{hr ft°F}}$$

$$\rho_a = 0.0715 \frac{\text{lbm}}{\text{ft}^3}$$

$$V_a = 14.3 \frac{\text{ft}^3}{\text{lbm}_{\text{dry air}}}$$

Assume an air flux, G", of 1000 lbm/hr ft² and a water flux, L", of 1500 lbm/hr ft². From Equation 2, the air velocity through the tower, s_G, is 3.97 ft/s. For a 2-mm diameter droplet, the droplet terminal velocity, s_f, is 19.21 ft/s (from reference 4, Table 2a, page 48). Therefore, the relative velocity of the droplet with respect to the air, s_a, is 15.24 ft/s. The "dynamical function," Ψ, for a 2-mm droplet is 5.23 seconds (from reference 4, Table 12, page 71). Substituting s_a and Ψ, along with the physical constants listed, into Equation 1 yields a mass transfer characteristic of

$$\frac{Ka}{L''} = 0.061 \text{ ft}^{-1} \tag{6}$$

The calculation of the pressure drop per foot of rain zone, $dN/d\ell$, using Equation 5 yields

$$NV' = \frac{dN}{d\ell} = 1.56 \qquad \frac{\text{heads}}{\text{ft}} \tag{7}$$

Another method of calculating the mass transfer and pressure drop within the rain zone is found in basic mass transfer and fluid dynamics theory. In mass transfer, the dimensionless group that contains the bulk mass transfer coefficient is the Sherwood number,

$$Sh = \frac{Ka\ D}{\rho_a D_{AB} A_v} \tag{8}$$

The term A_V in Equation 8 is defined as the droplet surface area per unit volume and is computed in terms of the droplet flux, Φ_d, as

$$A_V = \frac{A_s\ \Phi_d}{v_i} \qquad ft^{-1} \tag{9}$$

where v_i is the instantaneous velocity of the droplet. The Sherwood number correlation used in this analysis is [5],

$$Sh = 2 + (0.4\ Re^{1/2} + 0.06\ Re^{2/3})\ Sc^{0.4} \tag{10}$$

The Reynolds number found in Equation 10 is defined as

$$Re = \frac{V_R\ \ \rho D}{\mu} \tag{11}$$

The velocity in Equation 11 is computed by,

$$V_R = \sqrt{(u_a^2 - u_d^2)\ + (v_a^2 - v_d^2)} \qquad \frac{ft}{s} \tag{12}$$

The mass transfer characteristic, Ka/L", of the falling droplets in the rain zone can now be found by dividing both sides of Equation 8 by L" and solving for Ka/L".

The pressure drop associated with the droplets falling in the rain zone is computed based on the Lagrangian Equations of Motion. For a particular droplet, these equations are:

horizontal position, x,

$$x = \int_0^t u dt \qquad ft \tag{13a}$$

vertical position, y,

$$y = H + \int_0^t v\,dt \qquad \text{ft} \tag{13b}$$

horizontal velocity, u,

$$u = \int_0^t \frac{F_x g_c}{m_d}\, dt \qquad \frac{\text{ft}}{\text{s}} \tag{14a}$$

vertical velocity, v,

$$v = \int_0^t \frac{F_y g_c}{m_d}\, dt \qquad \frac{\text{ft}}{\text{s}} \tag{14b}$$

horizontal impulse, I_x,

$$I_x = \int_0^t F_x dt \qquad \text{lbf}\cdot\text{s} \tag{15a}$$

vertical impulse, I_y,

$$I_y = \int_0^t F_y dt \qquad \text{lbf}\cdot\text{s} \tag{15b}$$

The force, F, in Equations 14 and 15 is defined as,

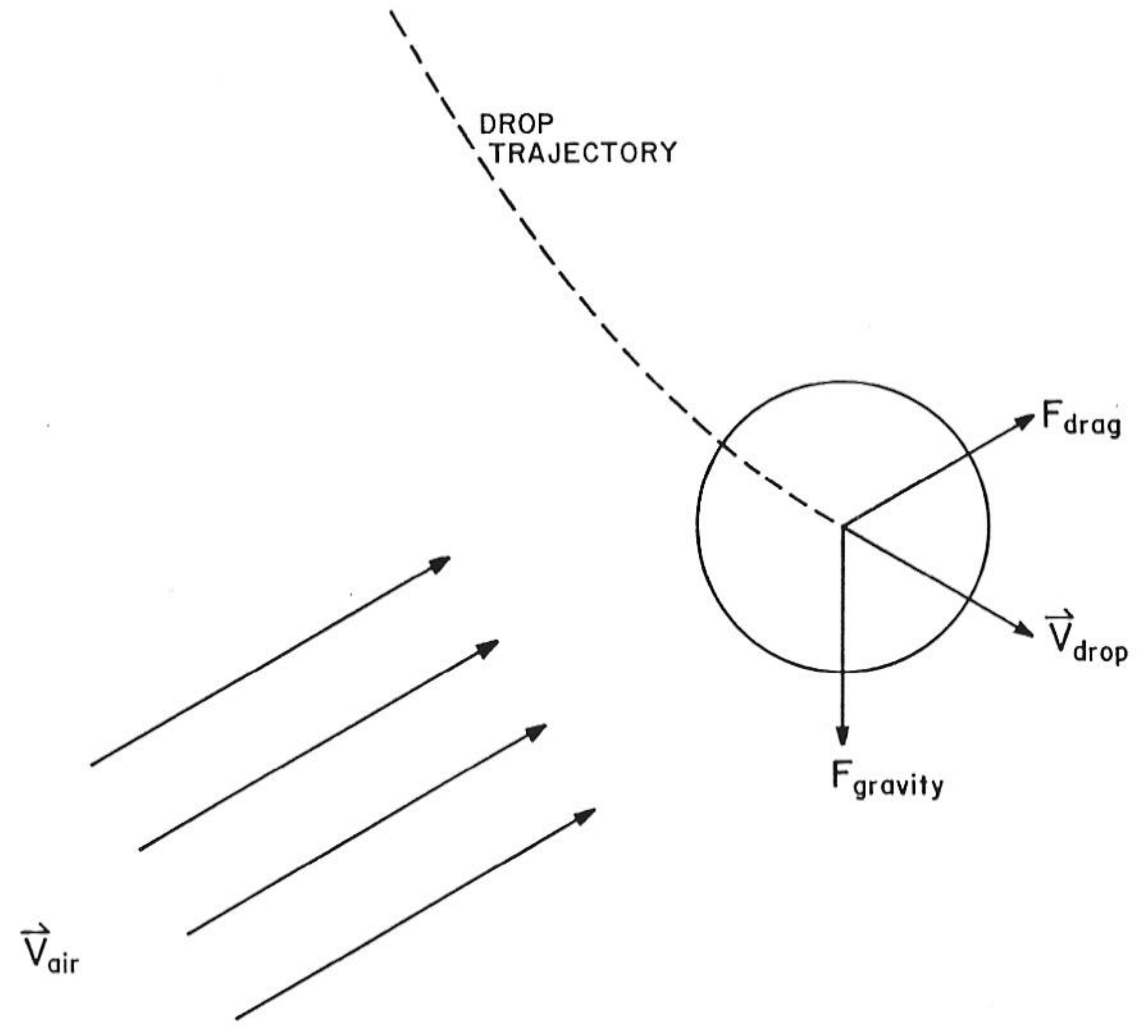
$$F_x = \frac{\pi\ D^2}{4}\ C_D\ \frac{\rho_a |u_a - u_d|(u_a - u_d)}{2g_c} \tag{16a}$$

and

$$F_y = \frac{\pi\ D^2}{4}\ C_D\ \frac{\rho_a |v_a - v_d|(v_a - v_d)}{2g_c} - \frac{(\rho_d - \rho_a)}{g_c}\ \frac{\pi D^3}{6} \tag{16a}$$

The pressure drop across the rain zone is found by integrating Equations 13, 14, and 15 to solve for the horizontal and vertical impulse. The pressure drop in terms of velocity heads lost per foot of rain zone is given by,

$$NV' = \frac{\sqrt{I_x{}^2 + I_y{}^2}\quad \phi_d}{H\quad \dfrac{\rho_d\ V_d{}^2}{2g_c}} \tag{17}$$



FORCE BALANCE ON A FALLING DROP

EXPERIMENTAL RESULTS AND COMPARISON

There have been several experimental studies done on the mass transfer and pressure drop from single droplets falling through air. However, a recent study conducted at the TVA Engineering Laboratory in Norris, Tennessee, [6] provides experimental data specifically on the transport phenomena occurring within the rain zone of a counterflow cooling tower. The test facility was designed to simulate the rain zone of a natural draft counterflow cooling tower where the air flows toward the center of the tower in a crossflow configuration. The test section is 15 feet long, 6 feet high and 4 feet wide with a maximum water flux of 2085 $lbm/hr \cdot ft^2$ and a maximum air flowrate of 15,000 cfm. The parameters measured during the experiments were the air and water flowrates, the hot water temperature entering the test section, the cold water temperature leaving the test section and the inlet wet bulb temperature. Given this data, the mass transfer characteristic, Ka/L", can be computed. Also, the pressure drop across the test section was measured, thereby giving the number of velocity heads lost per foot of air travel, NV'.

The results of the initial 12 tests conducted at the TVA Rain Zone Facility are given in Figure 2. In addition to the experimental data, the results of an analysis using the theoretical Sherwood number correlation (Equation 10) for a crossflow orientation are given. The theoretical curves show the dramatic effect of droplet size on the mass transfer from the droplets. From the figure, it can be seen that a difference in droplet diameter of a few millimeters can produce an order of magnitude difference in the mass transfer characteristic. Accurate droplet sizes were not experimentally determined during the TVA rain zone testing. However, the theoretical curve shown in Figure 2 for 3-mm droplets compare well with the experimental data.

Figure 3 shows droplet mass transfer as a function of L"/G" for a counterflow orientation. No experimental data for counterflow mass transfer were available. Two sets of curves are shown in the figure: the solid curves were computed based on the analysis discussed by Lowe

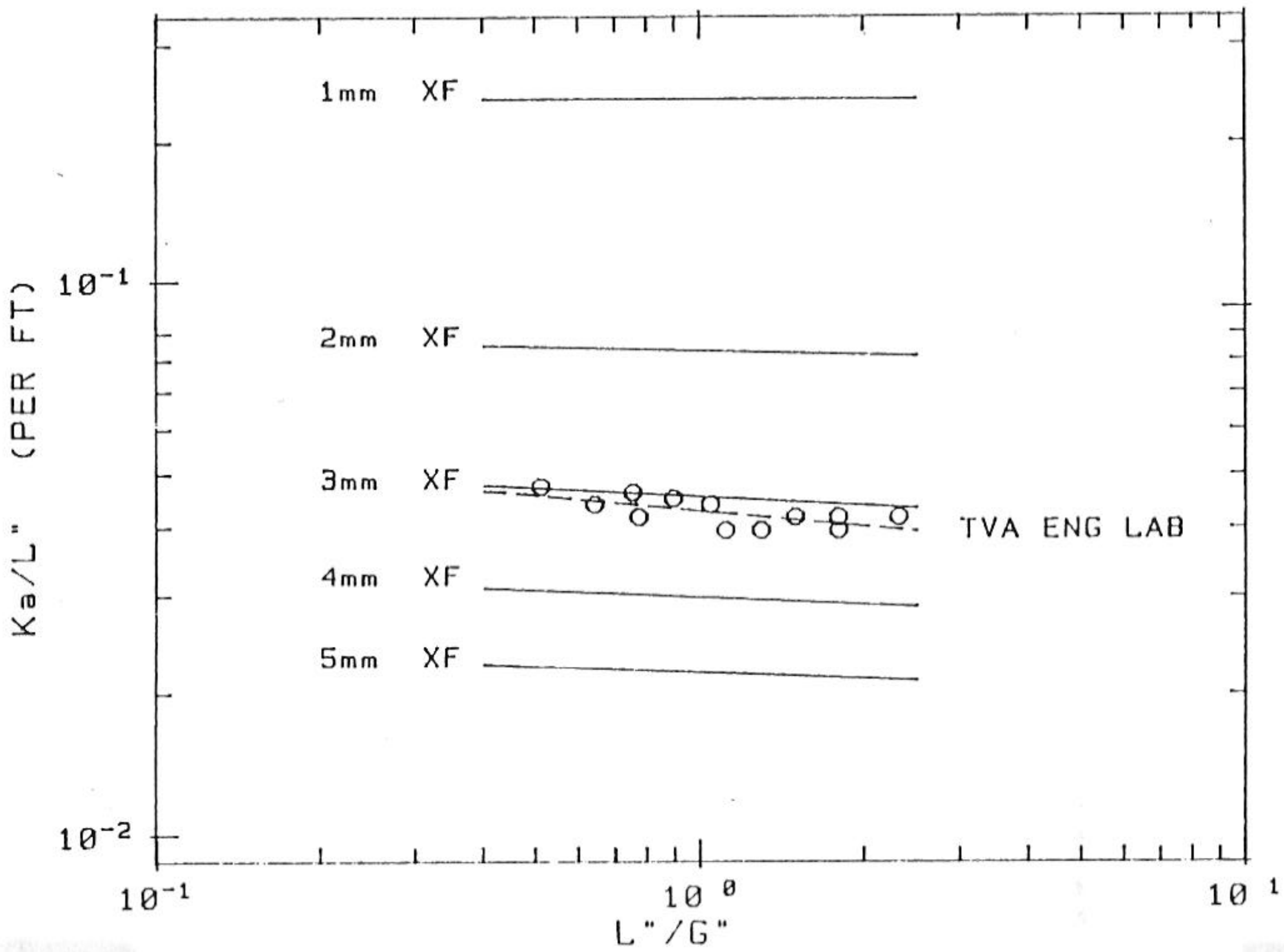


Figure 2. Crossflow Mass Transfer

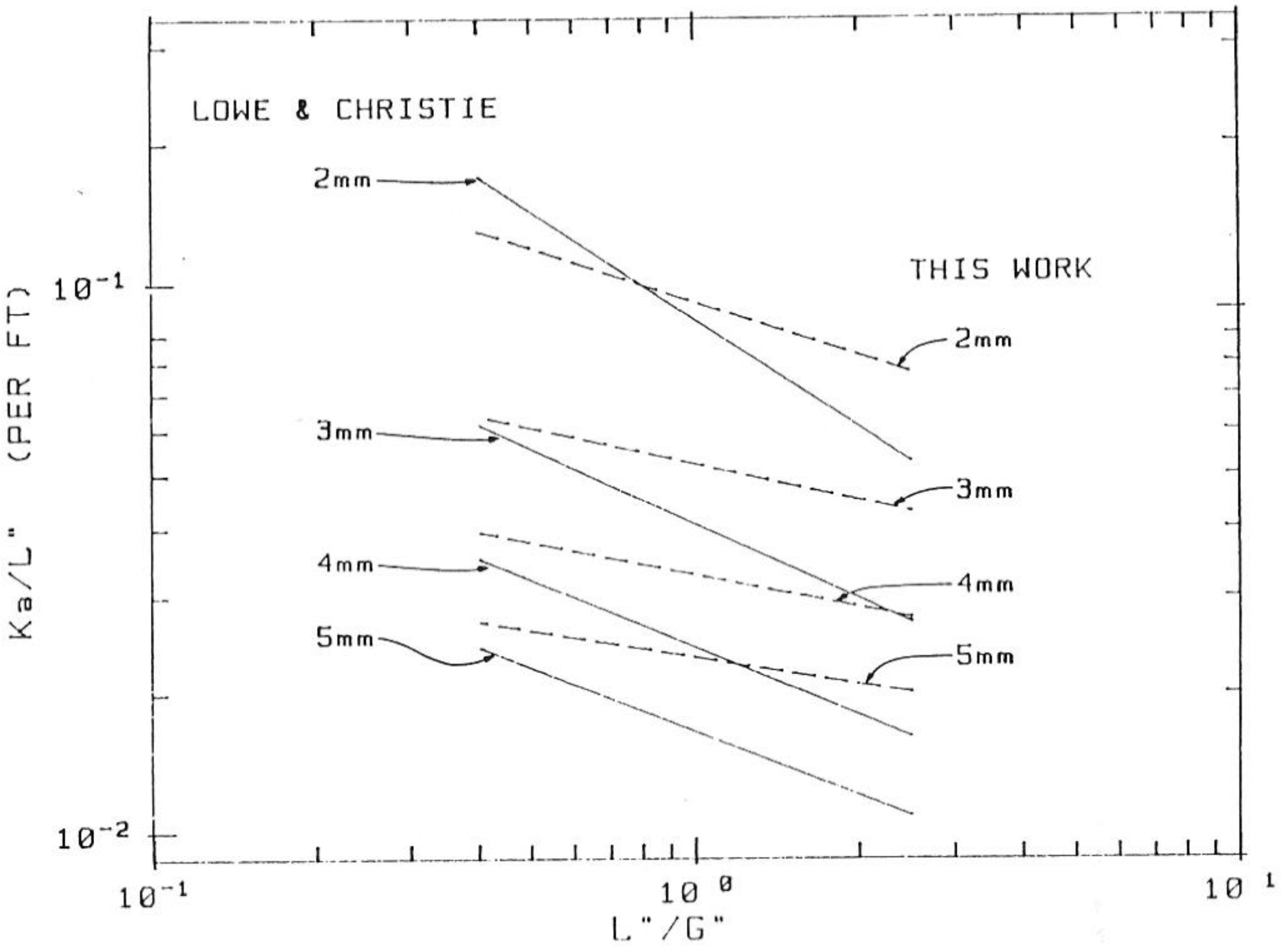


Figure 3. Counterflow Mass Transfer

and Christie [3], while the dashed curves were computed using Equations 8 through 17 as detailed above. As with the case of crossflow mass transfer, the dramatic effect of droplet size can be seen. The figure also shows the decrease in mass transfer coefficient with increasing water loading (increasing L"/G").

An interesting comparison of crossflow and counterflow rain zone mass transfer is shown in Figure 4. The curves shown in this figure were generated using Equations 8 through 17. As expected, for a given droplet diameter the mass transfer coefficient is higher for the counterflow orientation. Since the rain zone in a natural draft counterflow cooling tower is a combination of both counterflow and crossflow, for a given droplet size, the mass transfer characteristic for the rain zone would presumably lie between the counterflow and crossflow curves. Therefore, the curves shown in Figure 4 provide a theoretical upper and lower bound on the mass transfer actually occurring in the rain zone.

The pressure drop results for the TVA rain zone tests are given in Figure 5. The experimental data is plotted, as well as a series of theoretical curves, for various droplet diameters that are based on the drag force on the droplet (Equation 16). The dramatic effect of droplet diameter on the pressure drop is seen in the figure. The smaller droplets are more densely packed thereby producing a larger pressure loss. The theoretical curves for droplets between 2- and 3-mm compare well with the experimental data. This agrees fairly well with the comparison of droplet size for the mass transfer characteristic.

Figure 6 shows the pressure drop as a function of L"/G" for a counterflow configuration. Two sets of curves are shown in the figure representing analysis based on the method of Lowe and Christie and using Equations 8 through 17. Comparison of the counterflow results with the crossflow results indicates that the counterflow pressure drop is greater by as much as an order of magnitude.

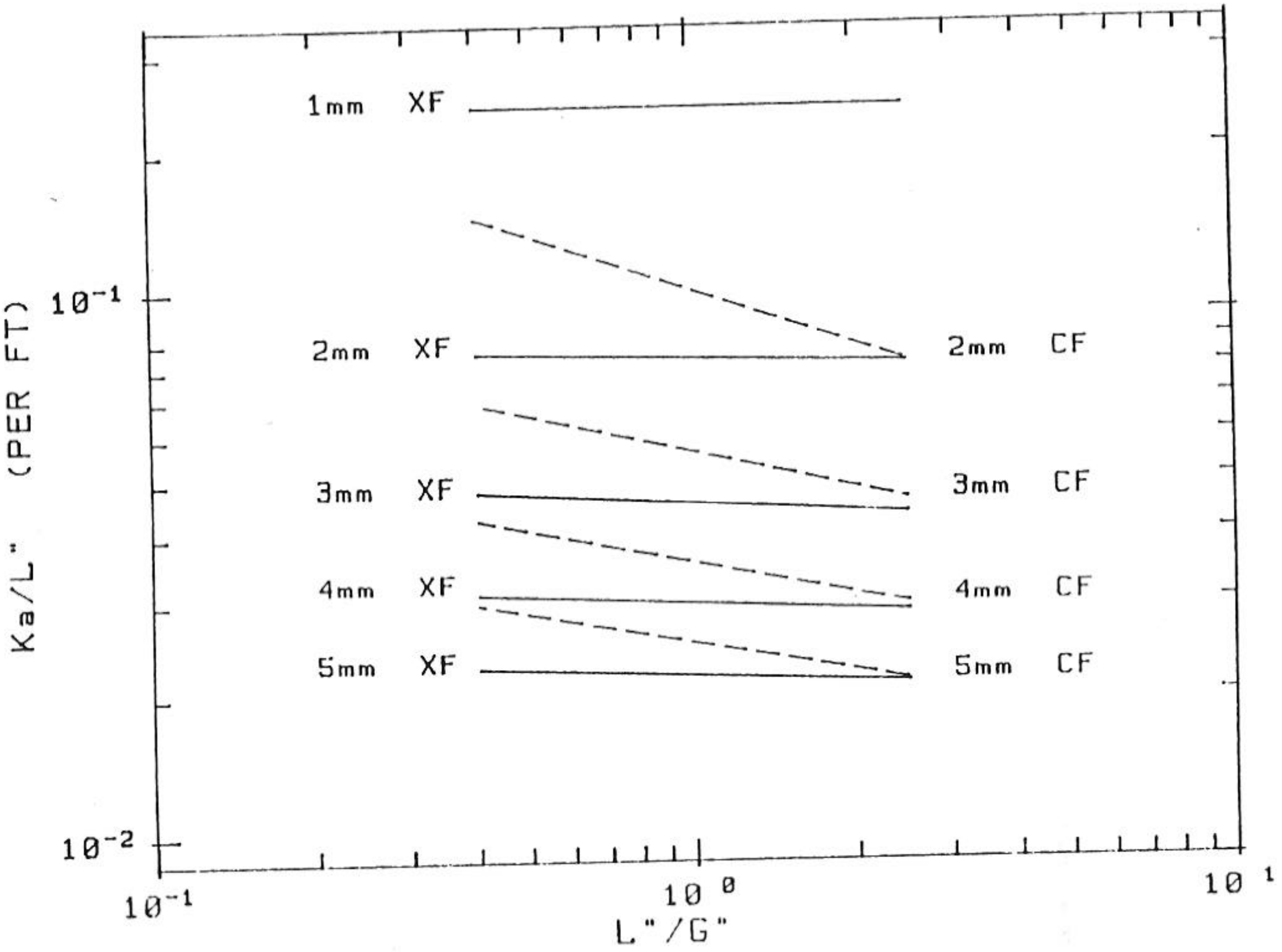


Figure 4. Crossflow and Counterflow Mass Transfer

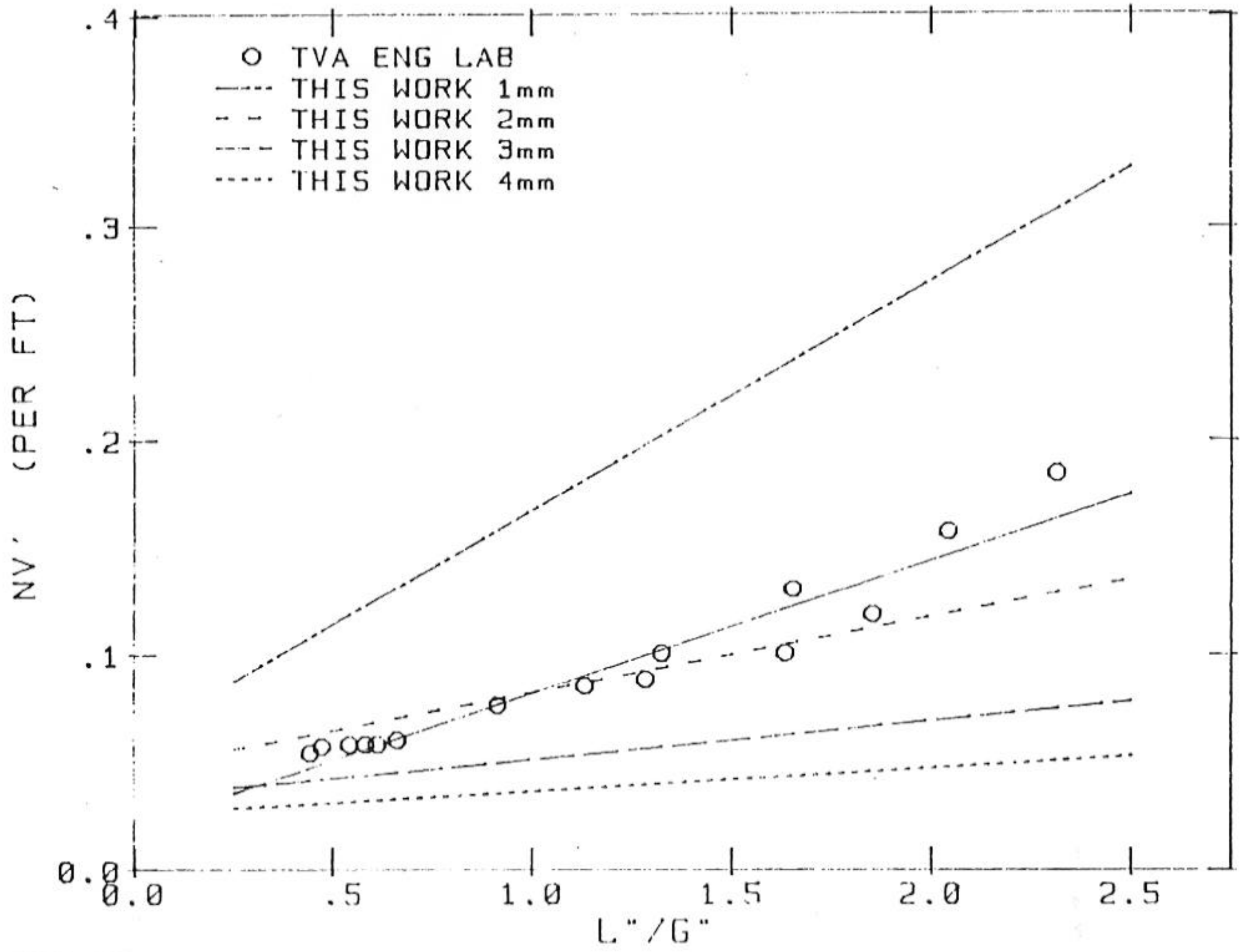


Figure 5. Crossflow Pressure Drop

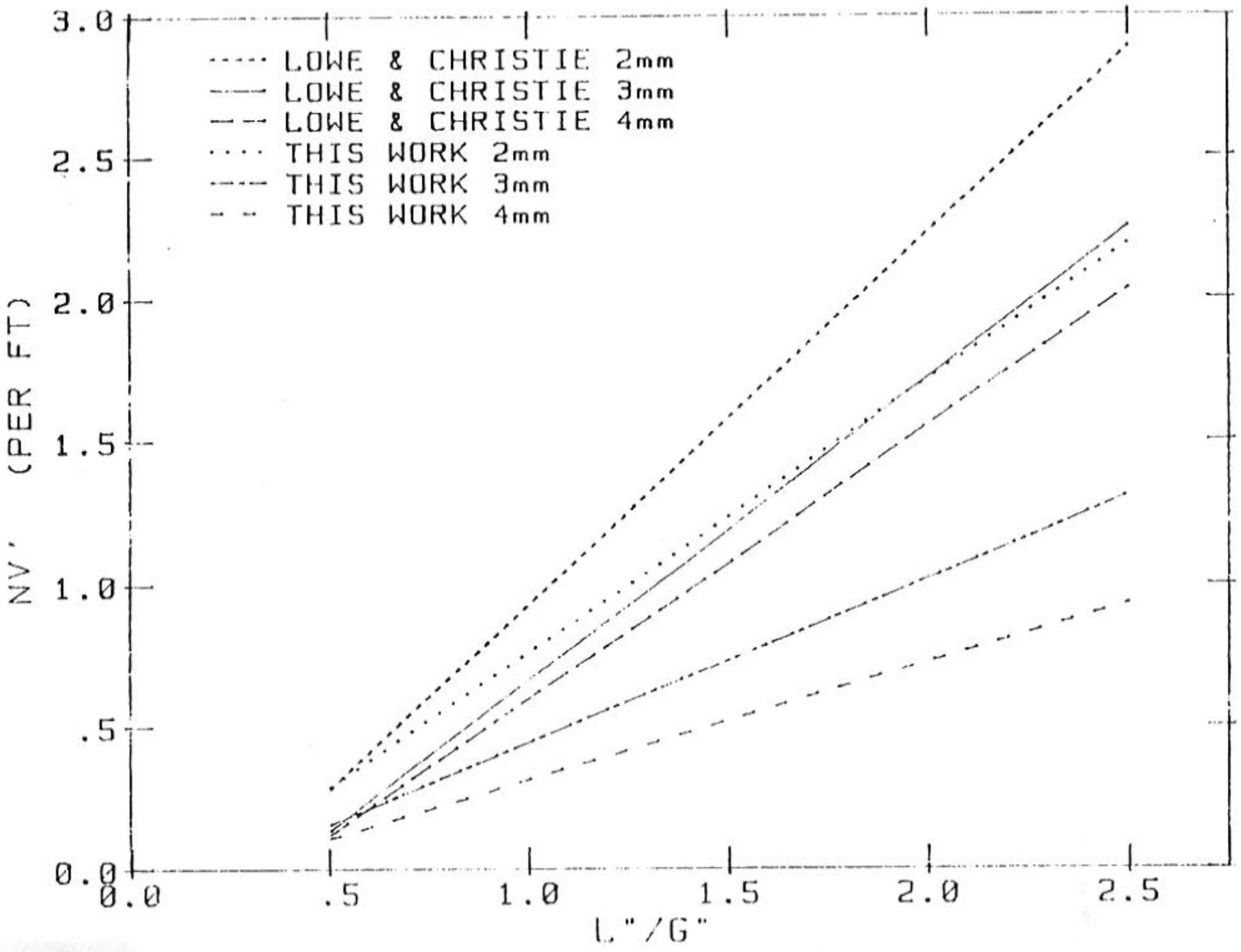


Figure 6. Counterflow Pressure Drop

SUMMARY

The present analysis can be applied to falling droplets with an orientation to the airflow ranging from crossflow to counterflow. This covers the range of orientations found in the rain zone of a natural draft counterflow cooling tower. The results of the analysis for mass transfer and pressure drop compare reasonably well with experimental data for crossflow and with the work of Lowe and Christie for counterflow. It is interesting to observe that the mass transfer characteristic is only weakly dependent on the orientation, as shown in Figure 3, but the pressure drop is strongly dependent (Figure 4 and 5). Certainly further analysis and experimentation are necessary, but this observation does raise the potential of using computer models to optimize the design of cooling towers to take advantage of this counterflow/crossflow phenomenon.

REFERENCES

1. Benton, Dudley J., and William R. Waldrop, "Computer Simulation of Transport Phenomena in Evaporative Cooling Towers," TVA Report Number WR28-1-900-141, March,1985.

2. Environmental Systems Corporation, "Examination and Comparison of Cooling Tower Component Heat Transfer Characteristics," presented at the IAHR Cooling Tower Workshop, Budapest, Hungary, 1982.

3. Lowe, H. J., and D. G. Christie, "Heat Transfer and Pressure Drop Data on Cooling Tower Packings and Model Studies of the Resistance of Natural Draught Towers to Airflow," International Division of Heat Transfer, Part V, pp 333-950, American Society of Mechanical Engineers, New York, 1961.

4. Nottage, H. B., and L.M.K. Boelter, "Dynamic and Thermal Behavior of Water Drops in Evaporative Cooling Processes," Transactions of the American Society of Heating and Ventilating Engineers, Vol. 46, pp 41-79, 1940.

5. Krieth, Frank, and W. Z. Black, Basic Heat Transfer, Harper and Rowe, 1980.

6. Missimer, Jack R., and C. Ann Brackett, "Results of Model Tests of Heat Transfer in the Rain Zone of a Counterflow Natural Draft Cooling Tower," TVA Report Number WR28-1-85-115, May 1985.

NOMENCLATURE

A_c - droplet cross sectional area, ft^2
A_s - droplet surface area, ft^2
A_v - droplet surface area per unit volume, ft^{-1}
C_D - drag coefficient
C_{Pa} - specific heat of air, Btu/lbm°F
C_{PL} - specific heat of water, Btu/lbm°F
D - diameter, ft
D_{AB} - mass diffusivity, ft^2/s
F_x - horizontal component of force on droplet, lb_f
F_y - vertical component of force on droplet, lb_f
G'' - air flowrate per area, lbm/hr ft^2
g - acceleration of gravity, ft/s^2
g_c - Newton's constant, 32.2 lbm ft/lbf$\cdot s^2$
H - height, ft
I - impulse, lb_f-sec
Ka - bulk mass transfer coefficient lbm/hr ft^2
L'' - water flowrate per area, lbm/hr ft^2
ℓ - length, ft
m_d - mass of droplet, lbm
NV' - pressure drop, heads/ft
P - pressure, lbf/ft^2
Re - Reynolds number
s_a - relative velocity of droplet, ft/s
s_f - droplet terminal velocity, ft/s
s_G - air velocity through rain zone, ft/s
Sc - Schmidt number
Sh - Sherwood number
t - time, s
u_a - horizontal component of velocity of air, ft/s
u_d - horizontal component of velocity of droplet, ft/s
V_a - specific volume of air, ft^3/lbm
v_R - relative velocity, ft/s
v_a - vertical component of velocity of air, ft/s

NOMENCLATURE

(continued)

v_d - vertical component of velocity of droplet, ft/s
v_i - instantaneous velocity, ft/s
x - horizontal position, ft
y - vertical position, ft
α - thermal diffusivity, ft^2/s
k_G - thermal conductivity of air, Btu/hr ft°F
k_L - thermal conductivity of water, Btu/hr ft°F
ϕ_d - droplet flux, drops/ft^2/s
ρ_a - density of air, lbm/ft^2
ρ_L - density of water, lbm/ft^3
μ - dynamic viscosity, lbm/ft•s
ψ - "dynamical function", s^{-1}