Modeling Nitrogen Supersaturation at Jennings Randolph

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ABSTRACT

Development and utilization of a numerical model of the nitrogen supersaturation process is an important component of the Jennings Randolph project. A literature review was conducted and the model developed by the U. S. Army Corps of Engineers (USACE) Waterways Experiment Station (WES) was found to be the most promising. This model was developed by D. A. Geldert, J. S. Gulliver, and S. C. Wilhelms and presented in a paper entitled, "Modeling Dissolved Gas Supersaturation Below Spillway Plunge Pools," which appeared in the May 1998 edition of the ***Journal of Hydraulic Engineering***.

INTRODUCTION

The WES model is based on elementary principles of fluid flow and mass transfer with empirical correlations providing closure. The model is zero-dimensional, that is, it does not subdivide the domain into elements and solve conservation equations within the elements. Application of the model, therefore, is limited to configurations similar to those used to develop the empirical correlations which provide model closure. The basic concept is that of a swarm of bubbles created by the plunging water rising to the surface and transferring nitrogen into the water in the stilling basin. Supersaturation can occur; because the bubbles are plunged downward into the water and experience greater than atmospheric pressure due to the hydrostatic pressure of the receiving water.

The Original WES Model

Geldert et al. (1998) report that first predictive model for dissolved gas levels downstream of a spillway was developed by Roesner and Norton (1971). They began with a simple mass transfer model that can be expressed as Equation 1:

$$C_d = C_s - (C_s - C_u) e^{-Kt} \tag{1}$$

where C_d is the downstream concentration, C_s is the saturation concentration, C_u is the upstream concentration, K is the mass transfer coefficient, and t is the residence time in the stilling basin. This equation forms the basis of the WES model.

Hibbs and Gulliver (1997) utilized Equation 2 in computing the effective saturation concentration, C_e:

$$C_e = C_s \left(1 + \frac{d_e \gamma}{P_a} \right) \tag{2}$$

where C_s is the saturation concentration (taken to be 100%), d_e is the effective bubble depth, γ is the specific weight of water, and P_a is the atmospheric pressure. Geldert et al. (1998) provide 3 field data sets: Ice Harbor, The Dalles, and Little Goose. The measured effective saturation concentrations and the values computed using Equation 2 are shown in Figure 1.

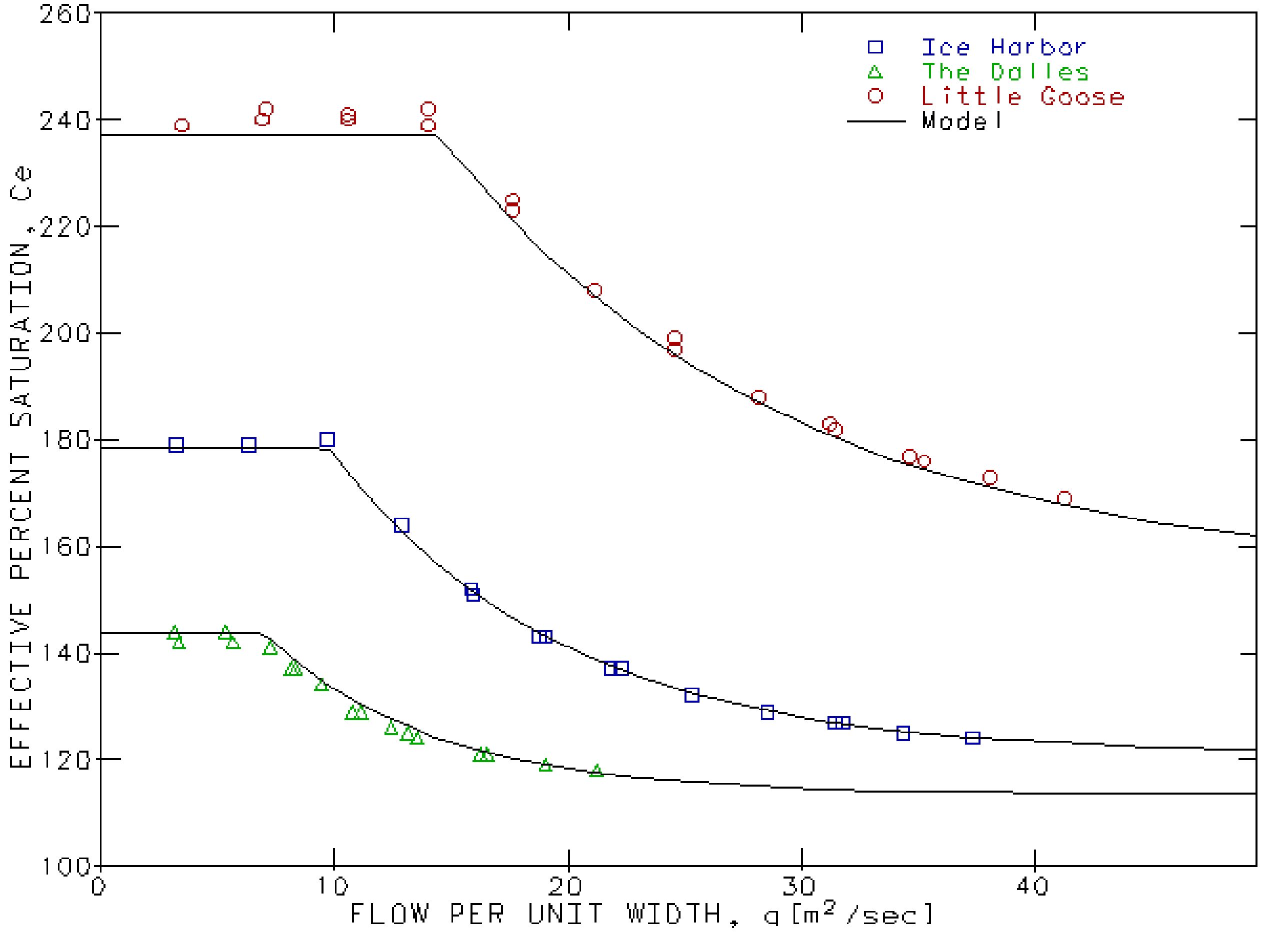

Figure 1. Computed and Measured Effective Saturation Concentration

The effective depth is computed from the bubble half-life depth (i.e., the length traveled over the half-life), $\boldsymbol{h_b}$, by Equation 3:

$$d_e = h_2 + (h_1 - h_2)\, e^{\left(1 - \frac{\beta h_b}{L_s}\right)} \quad \text{for} \quad \frac{\beta h_b}{L_s} > 1$$

$$d_e = h_1 \quad \text{for} \quad \frac{\beta h_b}{L_s} \leq 1$$

(3)where $\boldsymbol{\beta}$ is an empirical constant equal to

stilling basin. The bubble half-life depth is computed from the discharge per unit width, $\boldsymbol{q}$, and the bubble rise velocity, $\boldsymbol{v_r}$ (presumed be constant at 0.25 meters/second), by Equation 4.

$$h_b = \frac{q}{v_r} \ln(2) \tag{4}$$

Geldert et al. (1998) reasoned that the mass transfer included a bubble component into the water and a surface component out of the water. The rate of change of the concentration, $\boldsymbol{C}$, is then given by Equation 5:

$$\frac{dC}{dt} = K_L a_b (C_e - C) + K_L a_s (C_s - C)$$

(5)where $\boldsymbol{K_L}$ is the mass transfer coefficient, $\boldsymbol{a}$

$$C_d = C_e - (C_e - C_u)\left\{ e^{-(K_L a_b t_b + K_L a_s t_s)} + \frac{K_L a_s t_s}{K_L a_b t_b + K_L a_s t_s}\left(\frac{C_e - C_s}{C_e - C_u}\right)\left[1 - e^{-(K_L a_b t_b + K_L a_s t_s)}\right]\right\} \quad (6)$$

where t_b is the residence time for the bubbles and t_s is the exposure time for the surface transfer. Geldert et al. (1998) presumed that the combination $K_L a_s t_s$ would be a dimensionless constant on the order of 1 for any particular application.

The void fraction, φ, is computed using Equation 7

$$\phi = \frac{v_j \lambda}{v_j \lambda + q} \quad (7)$$

where λ is an empirical constant on the order of 0.2 meters and v_j is the effective velocity of the plunging jet of water. Geldert et al. (1998) did not provide a means of obtaining v_j, simply stating that this was "computed by a standard water surface profile technique." Geldert et al. (1998) used the void fraction and an empirical correlation to obtain the dimensionless bubble transfer group, $K_L a_b t_b$, given by Equation 8.

$$K_L a_b t_b = \alpha\phi \frac{(1-\phi)^{1/2}}{\left(1-\phi^{5/3}\right)^{1/4}} W_e^{3/5} R_q^{2/3} S_c^{-1/2} R_r^{-1} \quad (8)$$

where α is an empirical constant on the order of 1, W_e is the Weber number (Equation 9), R_q is the Reynolds number for the flow (Equation 10), S_c is the Schmidt number for air/water (Equation 11), and R_r is the Reynolds number for the rising bubbles (Equation 12).

$$W_e = \frac{\rho q^2}{\sigma d_j} \quad (9)$$

where ρ is the density of water, σ is the surface tension, and d_j is the effective depth of the plunging jet ($d_j = q/v_j$).

$$R_q = \frac{q}{\upsilon} \quad (10)$$

where υ is the kinematic viscosity of water.

$$S_c = \frac{\nu}{D} \quad (11)$$

where D is the air/water diffusion coefficient.

$$R_r = \frac{2 d_e v_r}{\upsilon} \quad (12)$$

These e

The Modified WES Model

The original WES model lacks an explicit calculation for the plunging jet velocity, v_j. In order to fill this gap in the model, a computer program was developed to "back out" the jet velocity implied by the data points for the 3 sites given in the WES report. These values were then compared to all of the dimensionless quantities, which can be formed from the site parameters. The best correlation obtained (r^2=0.92) is given in Equation 13 and illustrated in Figure 2.

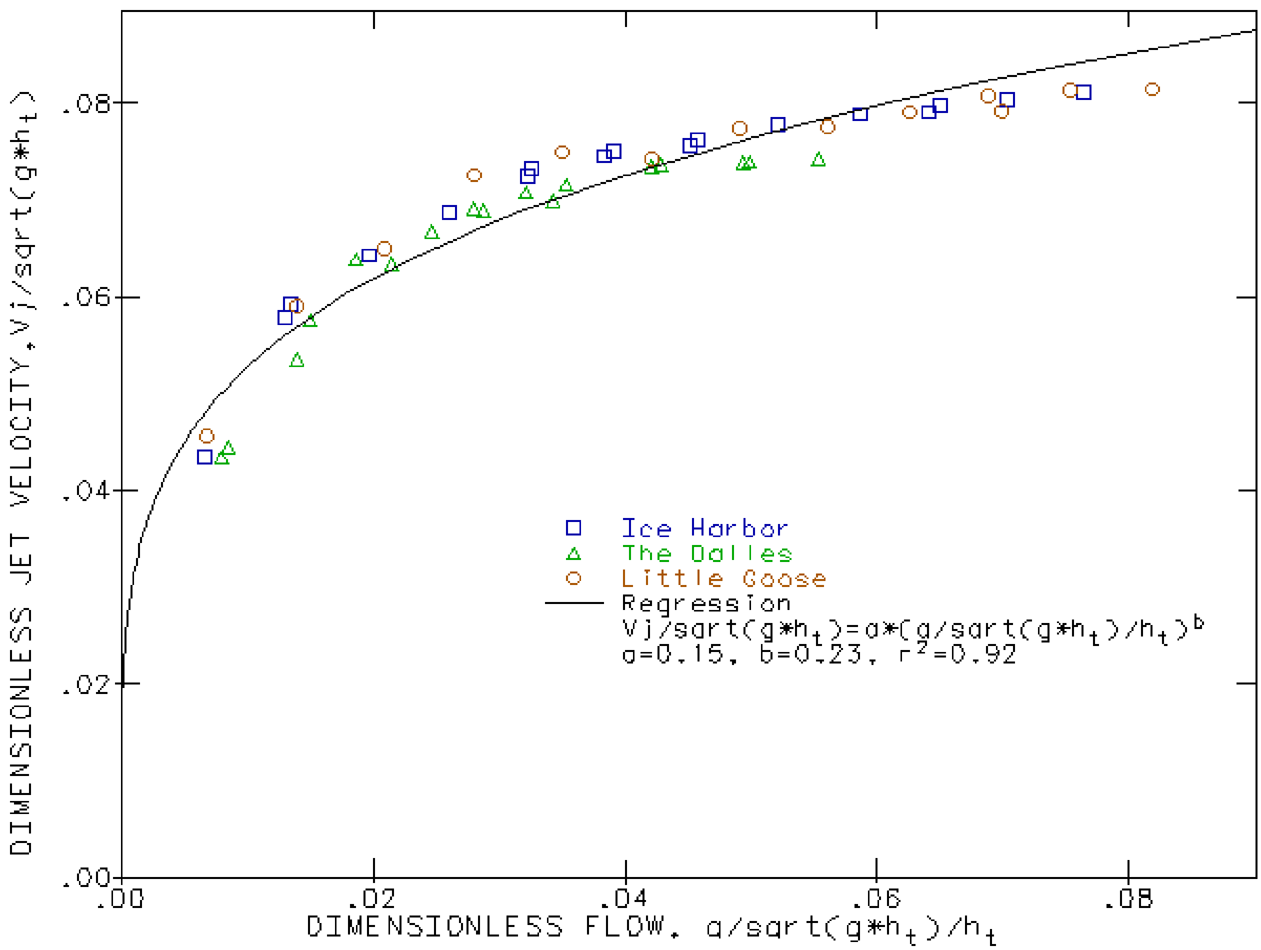

Figure 2. Dimensionless Correlation for Jet Velocity

$$\frac{v_j}{\sqrt{g\,h_t}} = 0.15\left(\frac{\frac{q}{h_t}}{\sqrt{g\,h_t}}\right)^{0.23} \qquad (13)$$

where $\boldsymbol{g}$ is the gravitational acceleration and $\boldsymbol{h_t}$ is the total (effective) head.

The bubble transfer group, $\boldsymbol{K_L a_b t_b}$, can then be computed from Equation 8 and this correlation for the jet velocity (Equation 13). The results are illustrated in Figure 3.

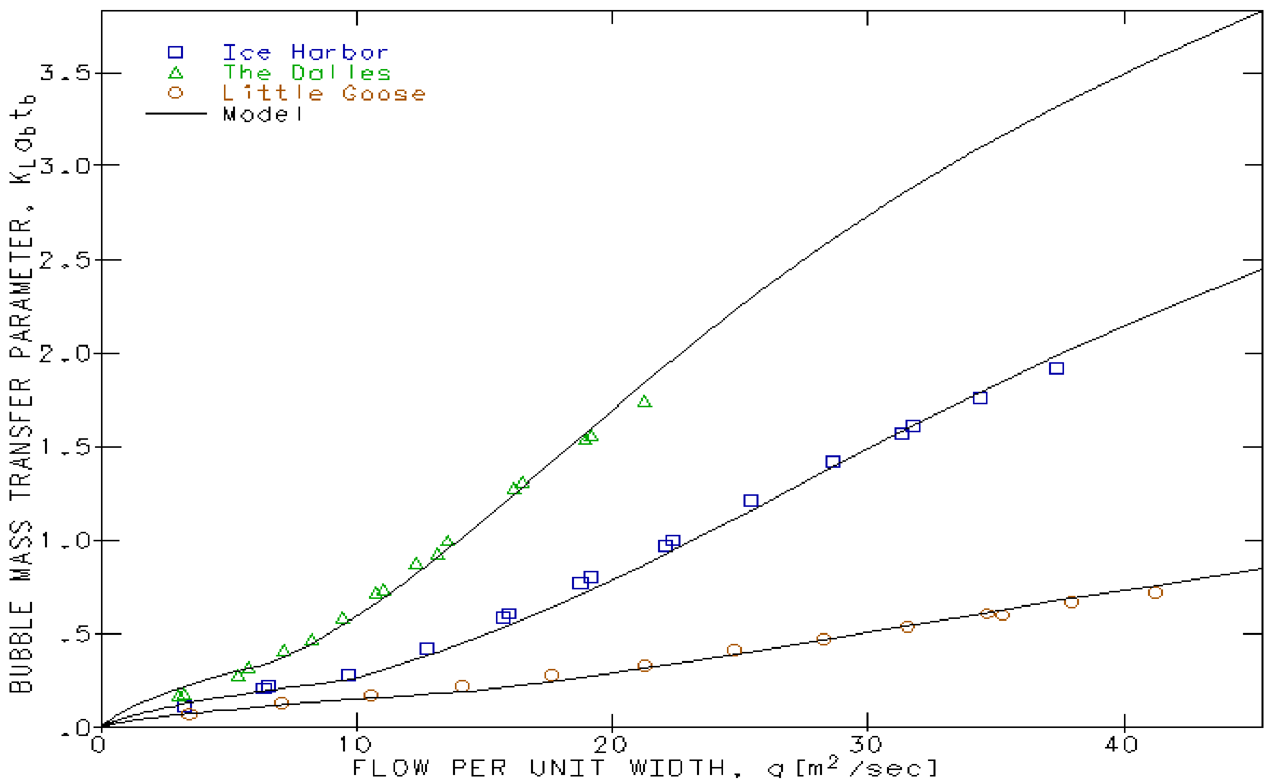

Figure 3. Bubble Transfer Group

This same model can be applied to the Jennings Randolph site. The data and model results for all 4 sites are illustrated in Figure 4.

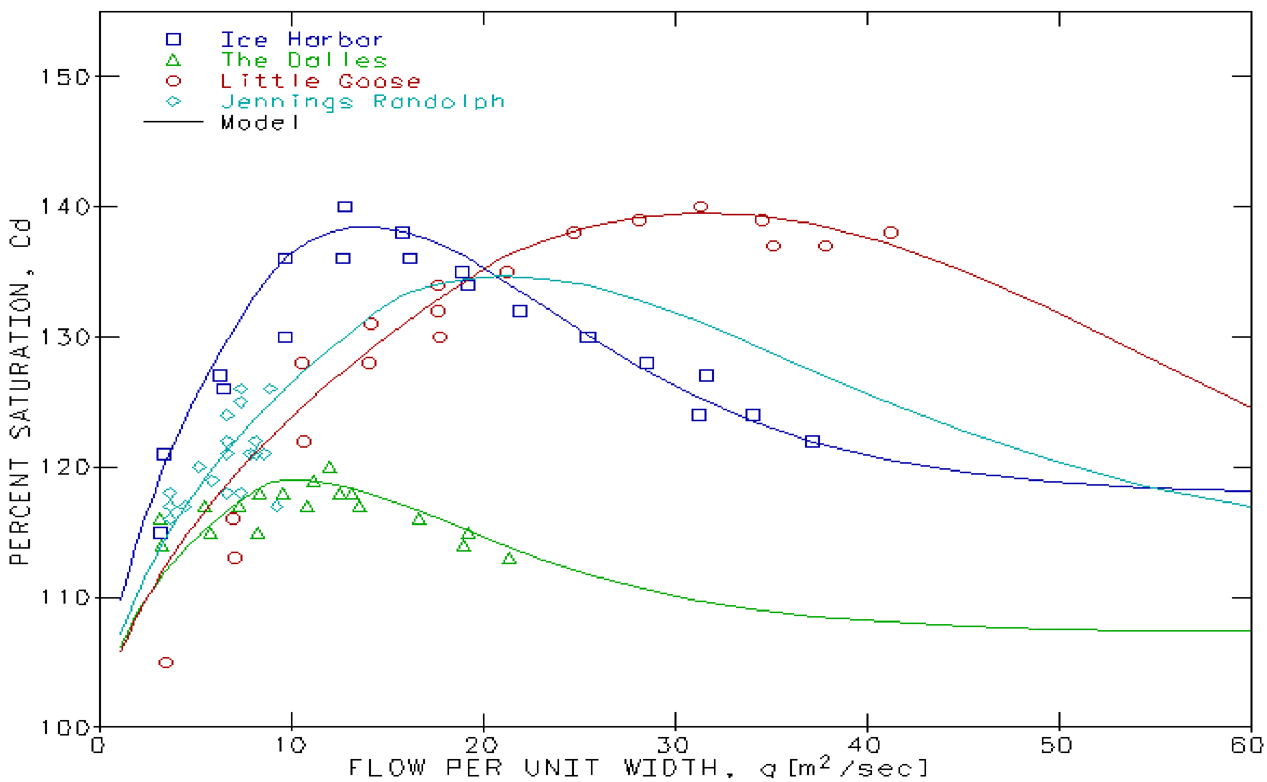

Figure 4. Data and Model Results for 4 Sites

Model Limitations

As stated previously, this is a zero-dimensional empirical model. The Modified WES Model is significantly different than a one-, two-, or three-dimensional finite difference or finite element model in which the domain is subdivided into computational cells. ***Any direct applications of this model are limited to the variables which appear in the various equations***, for instance, a single value must represent the depth of the stilling basin, h_s. If the depth of the stilling basin changes significantly over its length, this model will only accommodate a single number for the average or effective depth. If diverters or partitions are present in the stilling basin, there is no direct way to account for these in this model. There is also no direct way to account for such things as turbulence enhancers. The equations could be modified to account for some changes in the basic configuration on which the model is based; but such modifications would require some theoretical basis and experimental data or some established scaling law. Large changes to the configuration, such as weirs, would require at least an additional model and are likely inherently incompatible with the Modified WES Model. ***The decision to use a zero-dimensional empirical model for this project was made during contract negotiations***; as a multi-dimensional model would have required a significantly greater budget. ***Any multi-dimensional modeling would be outside the current scope of work.***

Modular Modeling

The only way to incorporate the impact of structures such as weirs is to subdivide the domain into modules. The model must already be subdivided into the stilling basin module and the downstream module. The same equation for surface mass transfer is used for the stilling basin and the downstream modules; so this subdivision does not represent a second model. An aeration weir is modeled by inserting a weir module between the stilling basin and the downstream module. The coupling of these modules is limited to their impact on the inlet conditions to the next model, that is, the exit conditions from the stilling basin module become the inlet conditions to the weir module; and the exit conditions from the weir model become the inlet conditions to the downstream module.

The Downstream Module

The concentration downstream of the stilling basin is modeled using the surface mass transfer component only from the stilling basin model. This can be expressed as Equation 14. The agreement of the Downstream Module with field data is shown in Figure 5. This figure shows computed vs. measured concentrations. Data taken at Barnum are indicated by circles and data taken at Blue Hole are indicated by triangles. Points falling on the diagonal solid line would indicate exact agreement between the model and data. Points falling below the diagonal solid line indicate a model prediction less than the measured value. Points lying above the diagonal solid line indicate a model prediction greater than the measured value. The 95% confidence interval is indicated by the two diagonal dotted lines. This interval is a statistical measure of the accuracy of the Downstream Module and is equal to ±3.1%. This means that 95 out of 100 data points should be within 3.1% of the corresponding calculated value.

$$C_d = C_u + \left(C_s - C_u\right)\left(1 - e^{-K_L a_s t_s}\right) \qquad (14)$$

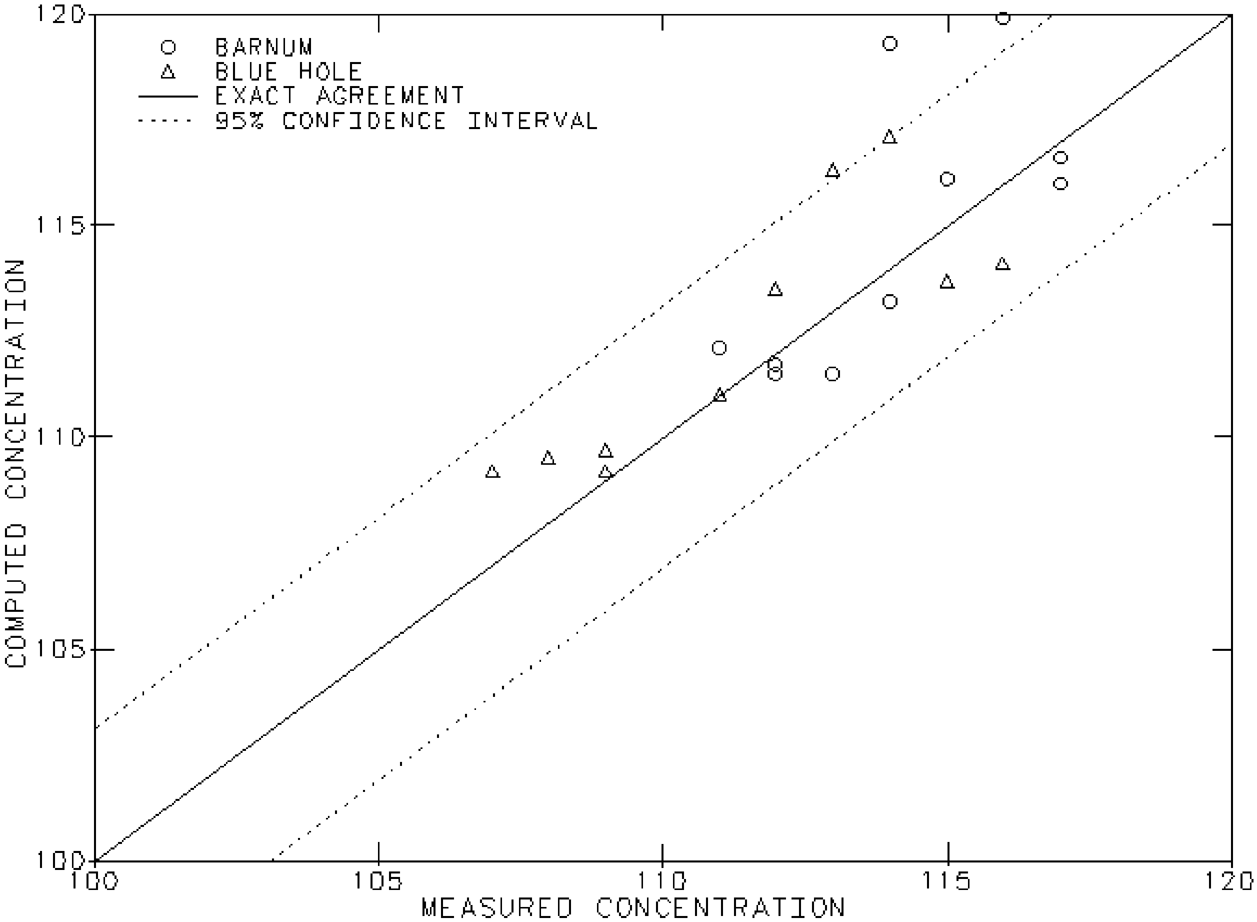

Figure 5. Agreement of Downstream Module and Field Data

The Weir Module

The aeration weir is also a zero-dimensional model. A single value of effectiveness is used. The effectiveness is expected to change with flow. The concentrations upstream and downstream of the aeration weir are related by Equation 15.

$$C_d = C_u + \varepsilon \left(C_s - C_u\right) \quad (15)$$

where $\boldsymbol{\varepsilon}$ is the effectiveness. The range of $\boldsymbol{\varepsilon}$ is zero to one, where zero would mean no effect and one would be complete approach to saturation.

RESULTS

The Model can now be used to predict the impact on downstream supersaturation of changes in the basic geometry of the stilling basin. Figure 6 shows the impact of increasing or decreasing the ***depth of the stilling basin*** by a factor of 2. Also shown in Figure 6 are the predicted concentrations for a discharge of 6300 CFS (i.e., the largest operating point in the field data set). The Model predicts a saturation at the downstream end of the stilling basin of 125.6%. The Model also predicts that this value would increase to 128.1% if the depth of the stilling basin were doubled and decrease to 121.5% if the depth of the stilling basin were half of its current value. Figure 6 also shows the maximum discharge such that the saturation is no more than 110%. The predicted value is approximately 1420 CFS, 1260 CFS, and 1210 CFS, respectively, for the 3 cases. While this change is not insignificant, it is insufficient to ameliorate the problem and still allow for a reasonable operating range.

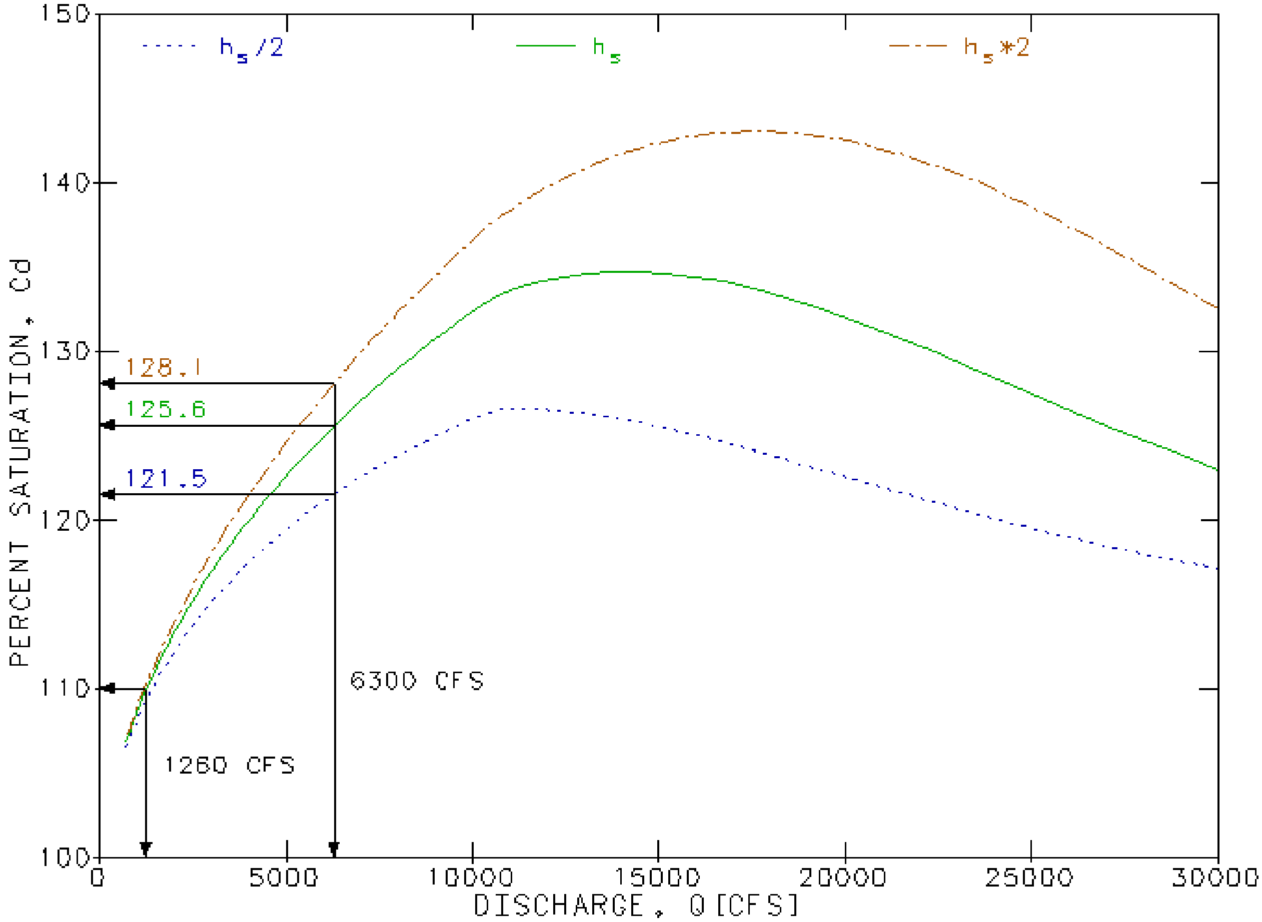

Figure 6. Impact of Stilling Basin Depth

Figure 7 shows the predicted impact of ***river depth at the end of the stilling basin*** on saturation. As seen in this figure model predictions indicate that any modification of the river depth at this point would provide no benefit within the operating range.

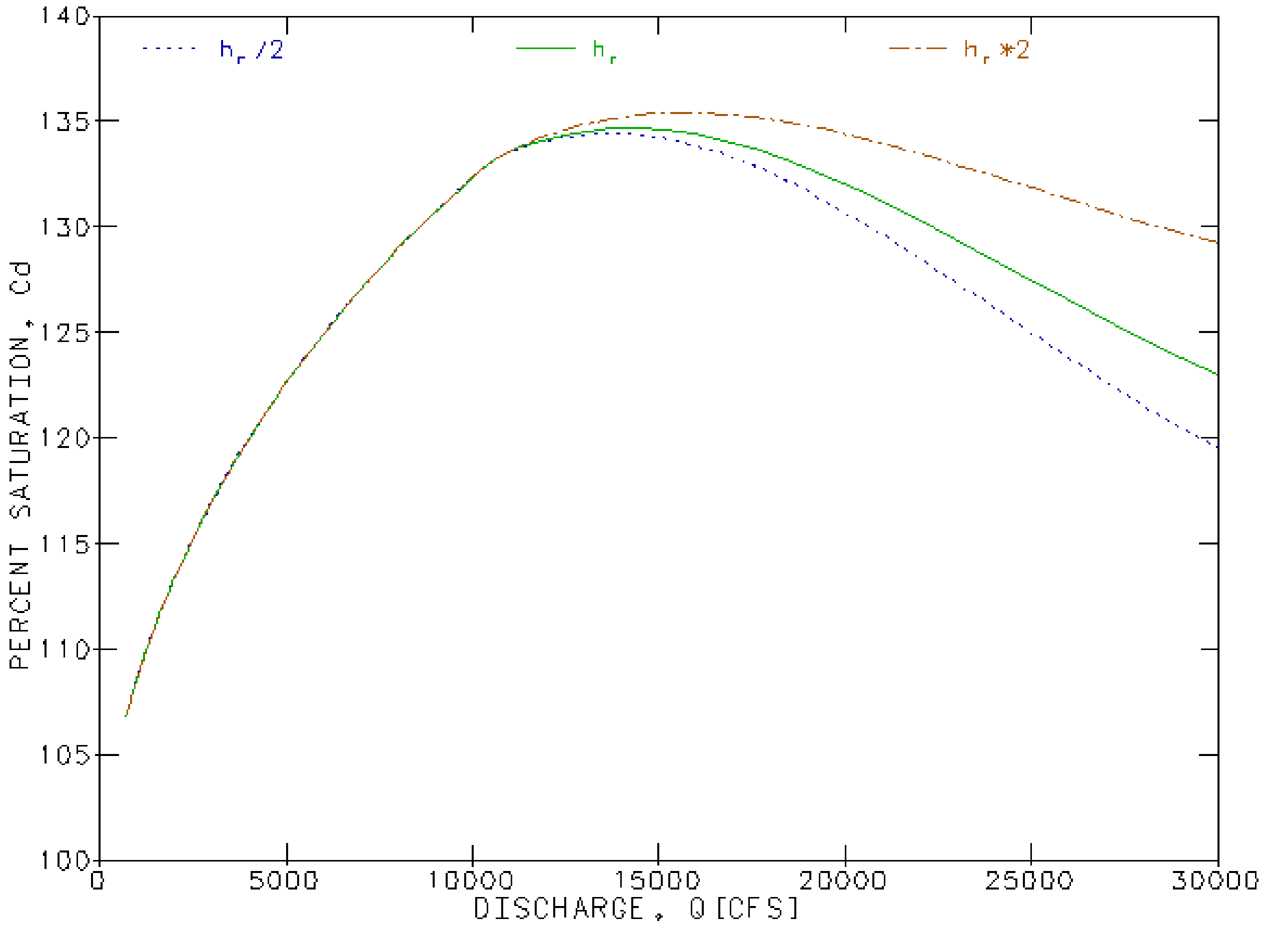

Figure 7. Impact of River Depth

Figure 8 shows the predicted impact of the ***length of the stilling basin*** on saturation. As was the case with a change in river depth, model predictions indicate that any modification of the stilling basin length would provide no benefit within the operating range.

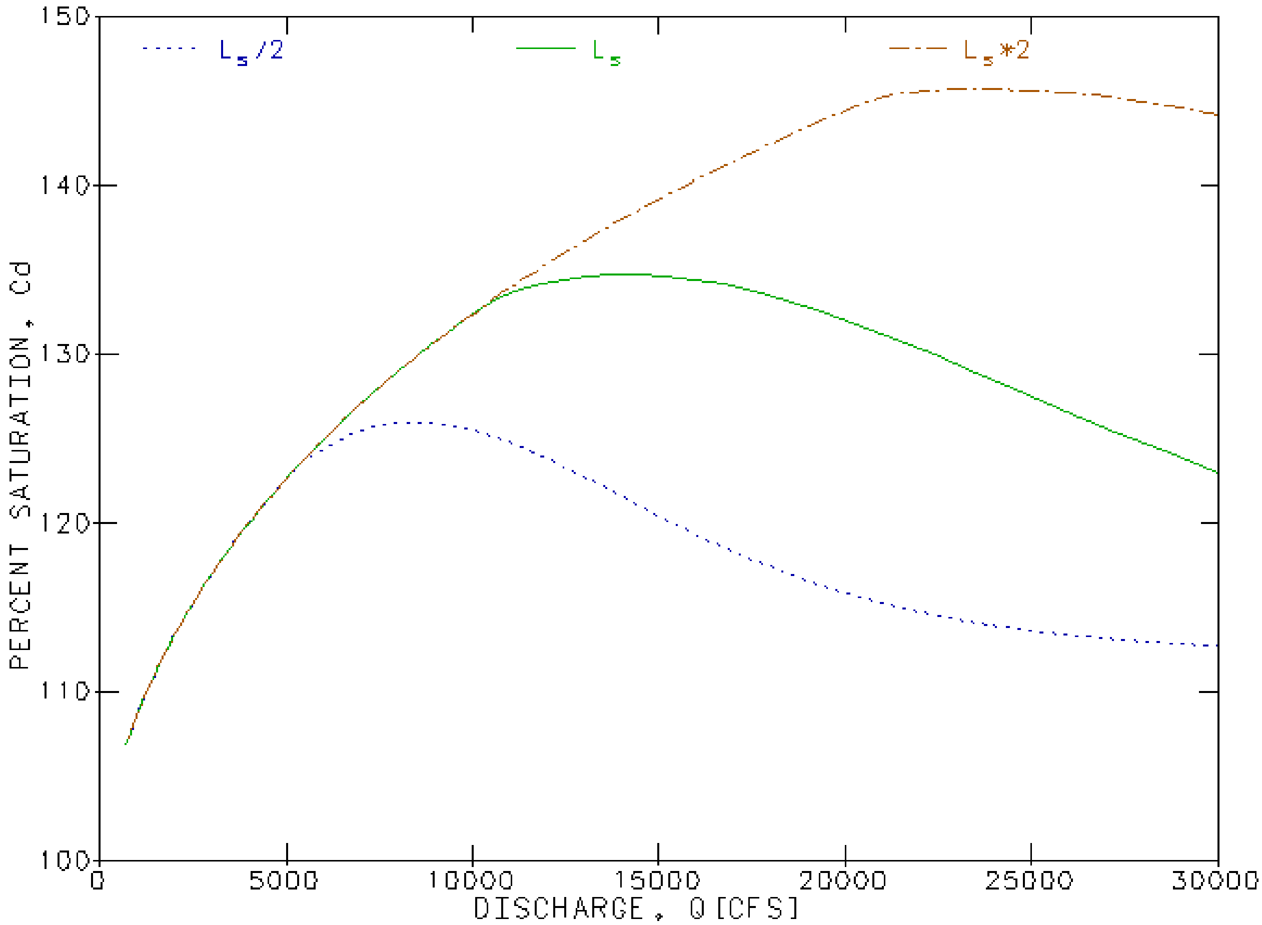

Figure 8. Impact of Stilling Basin Length

The Model does not accommodate a direct calculation for the ***impact of a deflector***; however, this impact is estimated by limiting the effective bubble depth, $\boldsymbol{d_e}$, which is an intermediate calculated parameter in the model, to some maximum plunge depth, $\boldsymbol{d_p}$. The energy dissipated in the stilling basin would be the same whether or not a deflector were present. If the plunging water were deflected, one would expect greater turbulence in the stilling basin. In order to make some accommodation for this increased turbulence, the mass transfer coefficient, $\boldsymbol{K_L}$, is increased by the same factor as the plunge depth is decreased. The results of these calculations are shown in Figure 9.

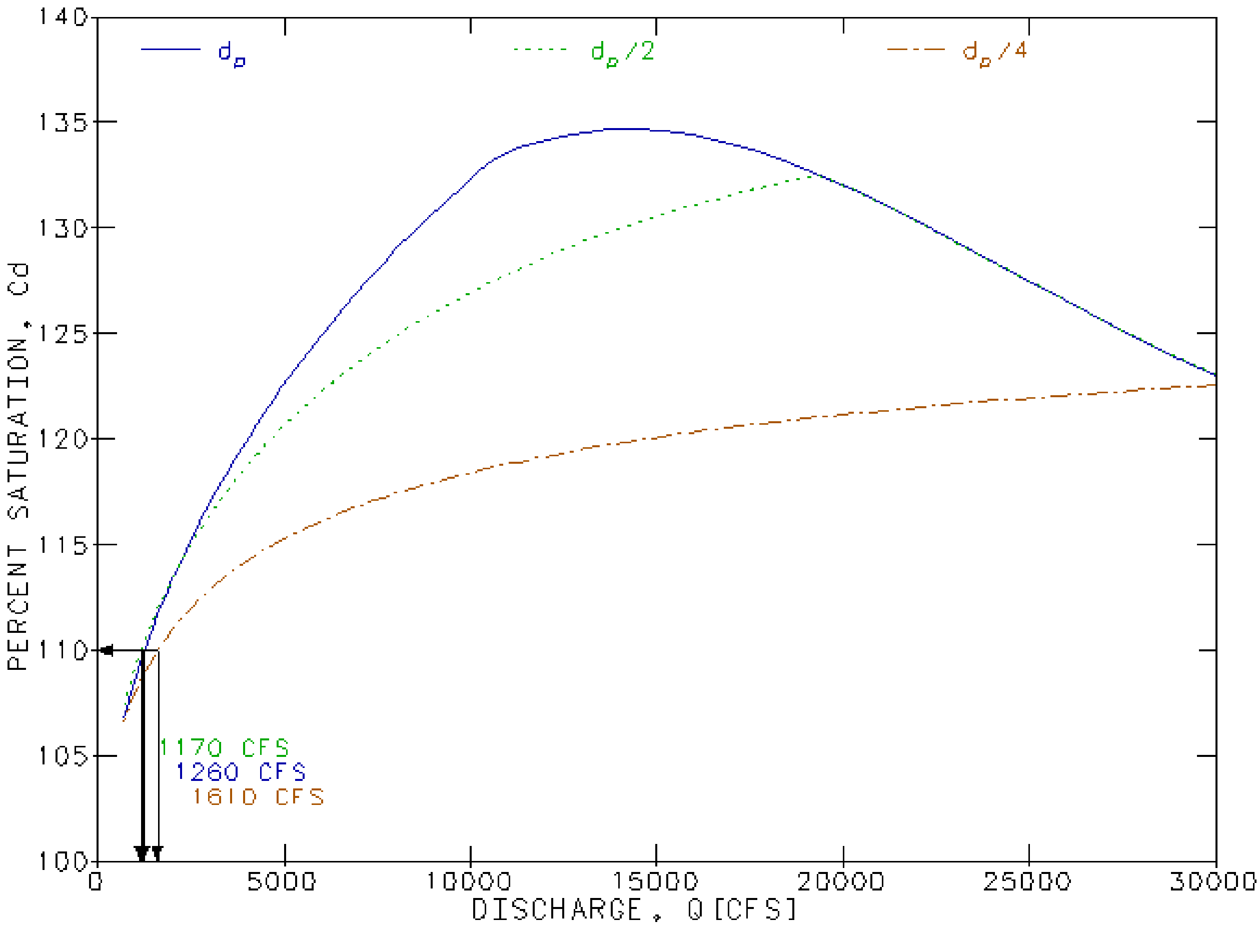

Figure 9. Impact of Limited Plunge Depth

These calculations must be considered an ***estimate and cannot be ascribed the same level of confidence*** as the basic model calculations. Figure 9 shows the model predicts that the greatest impact of limiting the plunge depth would be realized at discharges beyond the operating range. Figure 9 also shows that there may be a slight increase in supersaturation for low discharges were the plunge depth limited to one-half of its current value and a decrease in supersaturation were the plunge depth limited to one-fourth of its current value. This can be seen in Figure 9 by noting that the green dotted line (labeled $\boldsymbol{d_p}$/2) is slightly above the solid blue line (labeled $\boldsymbol{d_p}$); and the red chained line (labeled $\boldsymbol{d_p}$/4) is noticeably below the solid blue line where the lines intersect $\boldsymbol{C_d}$=110%. This seemingly anomalous impact at low discharge should not be considered significant; as this difference is less than the reasonable accuracy of the Model. The impact of limiting the plunge depth is clearly evident for larger discharges. Also shown on Figure 9 are the discharges corresponding to a saturation of 110%. The Model indicates that limiting the plunge depth to one-fourth of its current value would only increase the operating range from 1260 CFS to 1610 CFS, so as to not exceed 110% saturation. While this change is not insignificant, it is also insufficient to ameliorate the problem and still allow for a reasonable operating range.

A Composite Model is used to estimate the ***impact of an aeration weir*** on downstream saturations. The basic Model (Equations 2 through 13) plus the Weir Module (Equation 15) plus the Downstream Module (Equation 14) can be used to estimate the saturation at any distance downstream. Figure 10 shows the computed concentrations at a distance 3 miles downstream. The aeration weir effectiveness, $\boldsymbol{\varepsilon}$, is varied from 0 (or no weir), to 0.5 (a 50% effective weir), and 0.75 (a 75% effective weir). Of course, a completely effective weir ($\boldsymbol{\varepsilon}$=1) would eliminate all downstream supersaturation. Figure 10 shows that no weir is required ($\boldsymbol{\varepsilon}$=0) in order to keep saturation levels at or below 110% for discharges up to 3520 CFS. A 50% effective weir ($\boldsymbol{\varepsilon}$=0.5) would allow operation up to 8400 CFS without exceeding 110% at a distance 3 miles downstream. A 75% effective weir ($\boldsymbol{\varepsilon}$=0.75) would allow operation at any discharge without exceeding 110% at a distance 3 miles downstream. The Model indicates that operation above 2400 CFS will result in saturations exceeding 110% at Barnum unless an aeration weir is installed. This saturation would be reached at Blue Hole for discharges exceeding 3180 CFS. If a 50% effective weir were installed, the discharge could reach 3620 CFS before the saturation would exceed 110% at Barnum and could reach 7800 CFS before the saturation would exceed 110% at Blue Hole. A 75% effective weir would allow any discharge without exceeding a saturation of 110% at Barnum or Blue Hole.

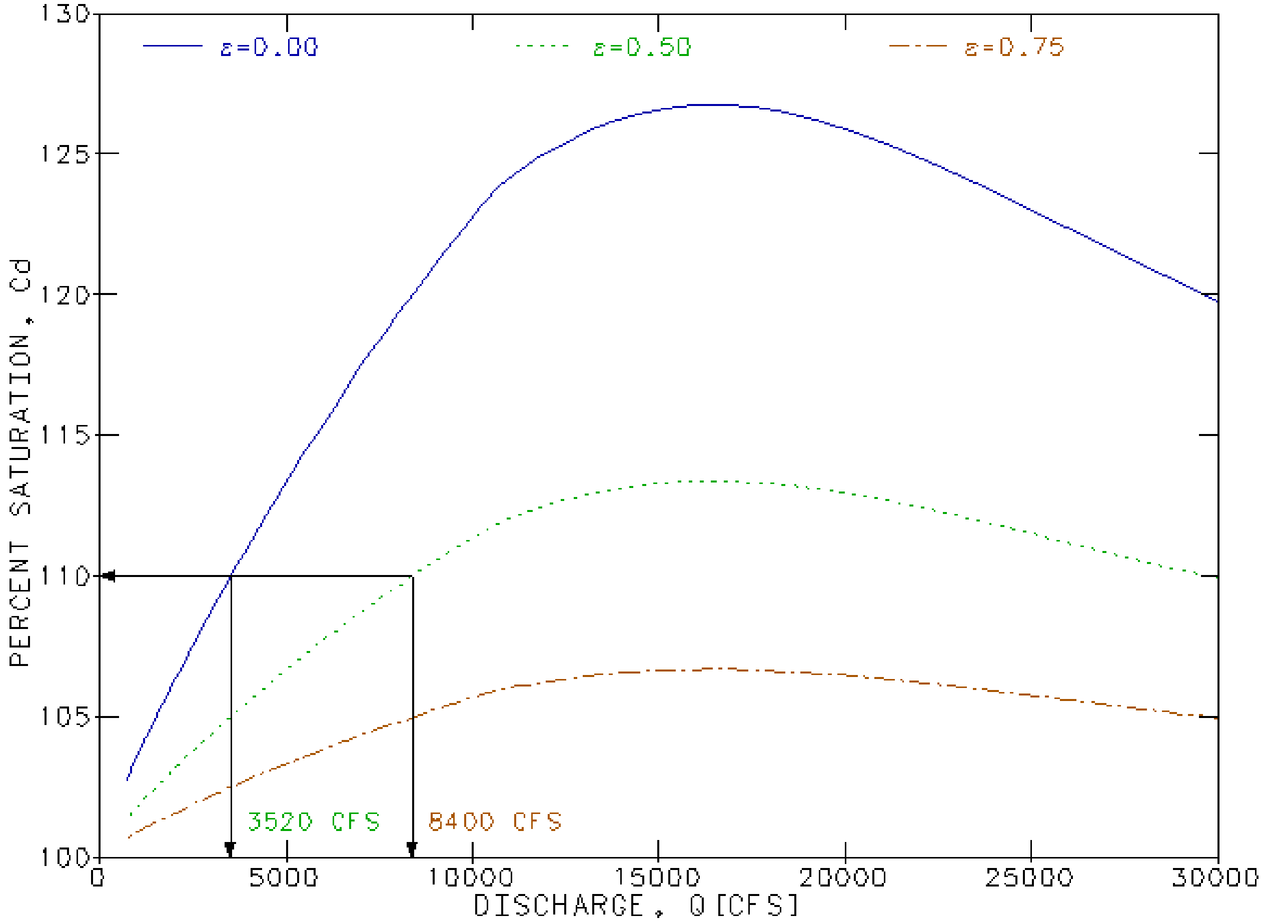

Figure 10. Computed Concentration 3 Miles Downstream

CONCLUSIONS

A modified version of the WES model for dissolved gas supersaturation below spillway plunge pools was used to predict the impact of various changes to the stilling basin and adjacent river bed on supersaturation. The Model was first calibrated using field data and then utilized to estimate the impacts. The Model predicts that only a slight benefit would be gained by reducing the depth of the stilling basin to one-half of its current value. The Model predicts that no benefit would be gained within the operating range by reducing the depth of the river at the end of the stilling basin to one-half of its current value. The Model predicts that no benefit would be gained within the operating range by doubling the length of the stilling basin. The Model predicts that only a slight benefit would be gained within the operating range by installing a deflector to limit the plunge depth to one-fourth of its current value. In summary, ***none of these four modifications would provide the desired benefit*** of reducing the saturation level at the end of the stilling basin to no more than 110% over any significant portion of the operating range. The Model predicts that a 50% effective aeration weir would allow discharges up to 2400 CFS without exceeding a saturation of 110% downstream at Barnum or Blue Hole. The Model predicts that ***a 75% effective aeration weir would allow any discharge*** without exceeding a saturation of 110% downstream at Barnum or Blue Hole. Nothing short of ***an 85% effective aeration weir would keep the saturation levels at or below 110% between the weir and Barnum***.

SYMBOLS

a_b bubble transfer area per unit volume [area/volume]
a_s surface transfer area per unit volume [area/volume]
C_d downstream concentration [%]
C_s saturation concentration [%]
C_e effective saturation concentration [%]
C_u upstream concentration [%]
D diffusion coefficient [length^2/time]
d_e effective bubble depth (Equation 3) [length]
d_j plunging jet depth [length] ($d_j=q/v_j$)
d_p maximum plunge depth [length]
g gravitational acceleration [length/time^2]
h_1 effective bubble depth in the stilling basin (Equation 3) [length] ($h_1=2h_s/3$)
h_2 ffective bubble depth in the river (Equation 3) [length] ($h2=h_r/2$)
h_b bubble half-life depth (Equation 4) [length]
h_r river depth [length]
h_s stilling basin depth [length]
h_t otal (effective) head [length]
K_L mass transfer coefficient [length/time]
L_s length of the stilling basin [length]
P_a atmospheric pressure (Equation 2) [force/area]
Q discharge [volume/time]
q discharge per unit width [volume/length/time]
R_q Reynolds number for the flow (Equation 10) [dimensionless]
R_r Reynolds number for the rising bubbles (Equation 11) [dimensionless]
S_c Schmidt number for air/water (Equation 12) [dimensionless]
t_b bubble transfer duration [time]

t_s surface transfer duration [time]
v_j plunging jet velocity (Equation 13) [length/time]
v_r bubble rise velocity (assumed to be 0.25 meters/second) [length/time]
W_e Weber number (Equation 9) [dimensionless]

greek
α empirical coefficient (Equation 8) [dimensionless]
β empirical coefficient (Equation 3) [dimensionless]
γ specific weight of water [force/volume]
ε effectiveness (Equation 15) [dimensionless]
λ empirical coefficient (Equation 7) [length]
υ kinematic viscosity of water [length^2/time]
ρ density of water [mass/volume]
σ surface tension [force/length]
φ void fraction (Equation 7) [dimensionless]

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